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# Introduction to AMC Code and Its Applications

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Motivation

#### Equations and Numerical Scheme

#### Model equation Eigenvalue solver

#### Benchmarks

Introduction

GAE TAE RSAE Ballooning mode Internal kink 'EAE' and 'NAE'

#### Applications

HL-2A experiments  $J_{\parallel 0}$  effects on RSAE

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Summary Future works

#### Appendix

Tearing mode in cylinder

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Motivation					
Why thi	s work?				

- 1. Provide a fast and easily used global eigenvalue solver to estimate the frequencies and mode structures of Alfvén eigenmodes (AEs) in experiments or large scale simulations.
- 2. Related to my work on ballooning mode (to benchmark GTC code).

AMC<sup>1</sup> (Alfvén Mode Code) is an eigenvalue code mainly (but not limited) aimed to study the Alfvén physics (continuum spectrums and eigenmodes) in tokamak.

<sup>&</sup>lt;sup>1</sup>Named by W. Chen.

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O Model equation	•000	0000000	0000000	00	000

### Vorticity equation

We solve the vorticity equation (shear Alfvén law)

$$\nabla \cdot \left(\frac{\omega^2}{v_A^2} \nabla_\perp \delta \phi\right) + \mathbf{B} \cdot \nabla \left(\frac{1}{B^2} \nabla \cdot B^2 \nabla_\perp Q\right) -$$

$$\nabla \left(\frac{J_{\parallel}}{B}\right) \cdot \left(\nabla Q \times \mathbf{B}\right) + 2 \frac{\kappa \cdot \left(\mathbf{B} \times \nabla \delta P\right)}{B^2} = 0,$$
(1)

 $\kappa = \mathbf{b} \cdot \nabla \mathbf{b}, \ Q = (\mathbf{b} \cdot \nabla \delta \phi) / B, \ \delta P = (\mathbf{b} \times \nabla \delta \phi \cdot \nabla P) / B, \ J_{\parallel} = \mathbf{b} \cdot \nabla \times \mathbf{B}.$ Eq.(1) holds for large aspect ratio ( $\epsilon = r/R \ll 1$ ) tokamak plasma to second order, and we have assumed low beta  $\beta \sim O(\epsilon^2)$ .

Further simplification: shifted circular geometry.

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Model equation					

Assuming  $\delta \phi = \sum \delta \phi_m(r) \exp(in\zeta - im\theta)$ , expanding Eq.(1) to  $O(\epsilon^2)$ , we obtain a coupled equation

$$\begin{aligned}
 L_{m,m-1}\delta\phi_{m-1} + L_{m,m}\delta\phi_m + L_{m,m+1}\delta\phi_{m+1} &= 0, \quad (2) \\
 L_{m,m} &= \frac{\partial}{\partial r} \Big[ \frac{(1 + 4\epsilon\Delta')}{v_A^2} \bar{\omega}^2 - k_m^2 - c_s^2 \Big] r \frac{\partial}{\partial r} + (k_m^2)' - \\
 \frac{m^2}{r} \Big\{ \frac{[1 - 4\epsilon(\epsilon + \Delta')]}{v_A^2} \bar{\omega}^2 - k_m^2 - c_s^2 - \bar{\kappa}_r \alpha/q^2 \Big\}, \quad (3) \\
 L_{m,m\pm 1} &= \bar{\omega}^2 \Big\{ \frac{\partial}{\partial r} \frac{(2\epsilon + \Delta')}{v_A^2} r \frac{\partial}{\partial r} - \frac{(\epsilon - \Delta')}{v_A^2} \frac{m(m \pm 1)}{r} \\
 \mp \frac{[\epsilon + (r\Delta')']}{v_A^2} m \frac{\partial}{\partial r} \Big\} - \Big\{ \frac{\partial}{\partial r} r \Delta' k_m k_{m\pm 1} \frac{\partial}{\partial r} - \frac{m^2}{r} (\epsilon + \Delta') k_m k_{m\pm 1} \\
 \mp m [\epsilon + (r\Delta')'] k_m k_{m\pm 1} \frac{\partial}{\partial r} \Big\} - \frac{m\alpha}{2q^2} \Big( \frac{m}{r} \mp \frac{\partial}{\partial r} \Big). 
 \tag{4}$$

 $ar{\omega} = \omega/(V_A/R_0), \ V_A = ig\langle v_A(r, heta)ig
angle, \ k_m = (n - m/q)$ , is the set of the s

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Eigenvalue solver					
Eigenval	ue solver				

Note: Different authors (e.g., Fu06, Breizman05, Berk92, Vlad99,  $\cdots$ ) may give different forms of  $L_{m,m}$  and  $L_{m,m\pm 1}$ . And, some of them may break the **self-adjointness** of the equation. Eqs.(2)-(4) are self-adjoint (all eigenvalues  $\omega^2$  are real). And, a term  $(k_m^2)'$  is added in  $L_{m,m}$  to support low m modes.

The above equation can be solved numerically for both continuum spectrums and eigenmodes. The continuum spectrums are obtained by setting the determinant of the coefficients of the secondorder derivative terms to zero. The eigenmodes are solved as a matrix eigenvalue problem  $\mathbf{AX} = \lambda \mathbf{BX}$ , with  $\omega^2 = \lambda$  and  $\mathbf{X} = [\cdots, \delta\phi_{m-1}, \delta\phi_m, \delta\phi_{m+1}, \cdots]^T$ .

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Eigenvalue solver

# Eigenvalue solver (cont.)

Eq.(2) supports a wide range of modes such as Alfvén eigenmodes (GAE, TAE, RSAE and more) and unstable internal kink mode as well as ideal ballooning mode (IBM).

Zero boundary conditions. Central difference discrete:  $\frac{df}{dr} = \frac{f_{j+1}-f_{j-1}}{2\Delta r}$ and  $\frac{d^2f}{dr^2} = \frac{f_{j+1}-2f_j+f_{j-1}}{\Delta r^2}$ .

The eigen matrix dimension is  $(N_m \times N_r)^2$ , where  $N_r$  is radial grid number and  $N_m = m_{max} - m_{min} + 1$  is number of *m* mode numbers. Sparse matrix is used to speed up. Typical run time is seconds or less [Other codes usually minutes or more]. Equations and Numerical Scheme

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#### GAE

Introduction

# Cylinder global Aflvén eigenmode

 $\rho = 1.0 - 0.98(r/a)^2$ ,  $q = 1.001 + 2.0(r/a)^2$ ,  $\beta = 0$ , n = 0, m = 2[PoP, 16, 072505].



Figure 1: GAE in AMC and KAEC codes,  $\omega_{GAE}^{AMC} = 1.3842$  and  $\omega_{GAE}^{KAEC} = 1.3843$ .

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TAE				00	000
Fu1989	TAF				

$$q = 1.0 + 1.0(r/a)^2$$
,  $\rho = 1.0$ , n=1 and  $R_0/a = 4$  [PFB, 1, 1949]



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 $\begin{array}{ll} \mbox{Figure 2:} & \mbox{TAE in Fu1989}, \ \omega_{\rm TAE}^{\rm Fu89} = 0.31, \ \omega_{\rm TAE}^{\rm NOVA} = 0.3127, \\ \omega_{\rm TAE}^{\rm KAEC} = 0.302, \ \omega_{\rm TAE}^{\rm AMC} = 0.303. \end{array}$ 

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#### TAE

#### Even and odd TAEs

 $q = 1.35 + 1.2(r/a)^2$ ,  $\rho = 1/[1 + 2.0(r/a)^2]$ , n=1 and  $R_0/a = 4$ 



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RSAE

# Reversed shear Alfvén eigenmodes

#### Deng2010 [PoP, 17, 112504]



Figure 4: RSAE in Deng2010,  $\omega_{\text{RSAE}}^{\text{GTC}} = 0.135$ ,  $\omega_{\text{RSAE}}^{\text{HMGC}} = 0.160$ ,  $\omega_{\rm RSAE}^{\rm AMC} = 0.147, \ \omega_{\rm RSAE}^{\rm accum} = 0.142.$ 

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O Ballooning mode	0000	0000000	0000000	00	000
Dallooning mode					

# Ballooning mode

#### Test run



Figure 5:  $\delta \phi_m(r)$ ,  $\delta \phi(r, \theta)$  for n = 20 mode and  $\gamma$  v.s. n.

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Internal kink					
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# Internal kink

Test run



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'EAE' and 'NAE'

# 'EAE' and 'NAE'

#### High order $(m \pm 2, 3, \cdots)$ gap AEs



For 'EAE' and 'NAE', the non-circular geometry is not a must!

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HL-2A experiments

# Sweeping RSAEs in HL-2A

#### Characterization of sweeping modes



A group of modes characterizes by down-sweeping frequency during NBI+ECRH and current ramp-up, and another group of modes characterizes by up-sweeping frequency before sawtooth crash during only NBI and current plateau.

>The toroidal mode numbers of two group modes are low, i.e. n=2-5, and poloidal mode number m=n.

The two group modes propagate poloidally parallel to the ion diamagnetic drift velocity and toroidally parallel to the plasma current direction in the laboratory frame of reference.

# Figure 6: W. Chen *et al.*, 13th IAEA-TM EP, 17-20 September 2013, Beijing, China.

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HL-2A experiments

# Sweeping RSAEs in HL-2A



Figure 7: W. Chen *et al.*, 13th IAEA-TM EP, 17-20 September 2013, Beijing, China.

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HL-2A experiments

# More data to be understood in HL-2A



Typical Instabilities Driven by Energetic Particles on HL-2A

Need more research on experiment and theory, such as From W. Chen measurement and calculation of mode structures

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HL-2A experiments

# More data to be understood in HL-2A (cont.)





From W. Chen

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$J_{\parallel 0}$ effects on RSA	E				
Parallel e	equilibrium current	effects on	the exist	ence of	
RSAE					

# $\begin{array}{l} \mathsf{RSAEs existence criterion [Berk2001 \ \mathsf{PRL}]} \ Q_{\mathrm{eff}} = Q_{\mathrm{f}} + Q_{\mathrm{tor}} + Q_{\mathrm{kink}} + \\ Q_{\mathrm{pressure}} + Q_{\mathrm{kinetic}} + ... > Q_{\mathrm{critical}} = 1/4 \end{array}$

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$J_{  0}$ effects on RS.	AE				
Without	$J_{\parallel 0}$				



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 $J_{\parallel 0}$  effects on RSAE

# With $J_{\parallel 0}$ (No RSAE)



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Summary					
Summar	V				

- 1. A fast and easily used global eigenvalue code is developed.
- 2. Good agreements with other codes.
- 3. Can be used as a tool for understanding the experiments and large scale simulations.

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Future works					
Future v	vorks				

1. Extending it to more complicated models (e.g., adding kinetic effects).

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2. Applying it for ballooning mode study, especially to benchmark GTC.

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Tearing mode in	cylinder				
Tearing	mode				
Reduced MHD equation for cylinder tearing mode					
	$\begin{cases} \partial_t \Psi = [\Psi, \phi] + \\ \partial_t U = [U, \phi] + \end{cases}$	$- \eta  abla_{\perp}^2 \Psi + \delta_{\perp}^2 - [\Psi, j_{arphi}] + \partial_{\perp}^2$	$\partial_{\varphi}\phi,\ _{\varphi}j_{\varphi}+ u abla_{\perp}^{2}U$	J.	(5)
U = V	$ abla_{\perp}^2 \phi$ , $j_arphi =  abla_{\perp}^2 \Psi$ .				

$$\begin{cases} [f,g] = \frac{1}{r} \left( \frac{\partial f}{\partial r} \frac{\partial g}{\partial \theta} - \frac{\partial g}{\partial r} \frac{\partial f}{\partial \theta} \right) = \frac{im}{r} \left( g \frac{\partial f}{\partial r} - f \frac{\partial g}{\partial r} \right), \\ \nabla_{\perp}^{2} = \frac{1}{r} \left( \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \right) + \frac{1}{r^{2}} \frac{\partial^{2}}{\partial \theta^{2}} = \frac{1}{r} \left( \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \right) - \frac{m^{2}}{r^{2}}. \end{cases}$$
(6)

Equilibrium relations:  $q^{-1} = -\frac{1}{r}\frac{d}{dr}\Psi_0$ ,  $j_0 = \nabla_{\perp}^2\Psi_0 = -\frac{1}{r}\frac{d}{dr}\frac{r^2}{q}$ ,  $s = \frac{r}{q}\frac{dq}{dr}$ ,  $U_0 = \phi_0 = 0$ .

Similar treatment will be used to extend AMC for toroidial tearing mode.

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Tearing mode in	cylinder				

### Tearing mode



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Tearing mode in cylinder

## Double tearing mode

