Introduction to AMC Code and Its Applications

Hua-sheng XIE (谢华生，huashengxie@gmail.com)

Department of Physics, Institute for Fusion Theory and Simulation,
Zhejiang University, Hangzhou 310027, P.R.China

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Ackn.: Y. Xiao, W. Chen (SWIP), L. M. Yu (ECUST), S. Wang, G. Y. Fu, F. Zonca, · · ·

Why this work?

1. Provide a fast and easily used global eigenvalue solver to estimate the frequencies and mode structures of Alfvén eigenmodes (AEs) in experiments or large scale simulations.

2. Related to my work on ballooning mode (to benchmark GTC code).

AMC[1] (Alfvén Mode Code) is an eigenvalue code mainly (but not limited) aimed to study the Alfvén physics (continuum spectrums and eigenmodes) in tokamak.

[1] Named by W. Chen.

Vorticity equation

We solve the vorticity equation (shear Alfvén law)

$$\nabla \cdot (\frac{\omega^2}{v_A^2} \nabla_\perp \delta\phi) + \mathbf{B} \cdot \nabla (\frac{1}{B^2} \nabla \cdot B^2 \nabla_\perp Q) - \nabla (\frac{J_\parallel}{B}) \cdot (\nabla Q \times \mathbf{B}) + 2 \frac{\kappa \cdot (\mathbf{B} \times \nabla \delta P)}{B^2} = 0, \tag{1}$$

$\kappa = \mathbf{b} \cdot \nabla \mathbf{b}$, $Q = (\mathbf{b} \cdot \nabla \delta\phi)/B$, $\delta P = (\mathbf{b} \times \nabla \delta\phi \cdot \nabla P)/B$, $J_\parallel = \mathbf{b} \cdot \nabla \times \mathbf{B}$. Eq.(1) holds for large aspect ratio ($\epsilon = r/R \ll 1$) tokamak plasma to second order, and we have assumed low beta $\beta \sim O(\epsilon^2)$.

Further simplification: shifted circular geometry.

Model equation

Assuming $\delta\phi = \sum \delta\phi_m(r)\exp(in\zeta - im\theta)$, expanding Eq.(1) to $O(\epsilon^2)$, we obtain a coupled equation

$$L_{m,m-1}\delta\phi_{m-1} + L_{m,m}\delta\phi_m + L_{m,m+1}\delta\phi_{m+1} = 0, \tag{2}$$

$$L_{m,m} = \frac{\partial}{\partial r}\left[\frac{(1+4\epsilon\Delta')}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2\right] r\frac{\partial}{\partial r} + (k_m^2)' - \frac{m^2}{r}\left\{\frac{[1-4\epsilon(\epsilon+\Delta')]}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2 - \bar{\kappa}_r\alpha/q^2\right\}, \tag{3}$$

$$L_{m,m\pm1} = \bar{\omega}^2\left\{\frac{\partial}{\partial r}\frac{(2\epsilon+\Delta')}{v_A^2}r\frac{\partial}{\partial r} - \frac{(\epsilon-\Delta')}{v_A^2}\frac{m(m\pm1)}{r} \mp \frac{[\epsilon+(r\Delta')']}{v_A^2}m\frac{\partial}{\partial r}\right\} - \left\{\frac{\partial}{\partial r}r\Delta' k_m k_{m\pm1}\frac{\partial}{\partial r} - \frac{m^2}{r}(\epsilon+\Delta')k_m k_{m\pm1} \mp m[\epsilon+(r\Delta')']k_m k_{m\pm1}\frac{\partial}{\partial r}\right\} - \frac{m\alpha}{2q^2}\left(\frac{m}{r} \mp \frac{\partial}{\partial r}\right). \tag{4}$$

$\bar{\omega} = \omega/(V_A/R_0)$, $V_A = \langle v_A(r,\theta)\rangle$, $k_m = (n - m/q)$.

Eigenvalue solver

Note: Different authors (e.g., Fu06, Breizman05, Berk92, Vlad99, $\cdots$) may give different forms of $L_{m,m}$ and $L_{m,m\pm1}$. And, some of them may break the **self-adjointness** of the equation. Eqs.(2)-(4) are self-adjoint (all eigenvalues ω^2 are real). And, a term $(k_m^2)'$ is added in $L_{m,m}$ to support low m modes.

The above equation can be solved numerically for both continuum spectrums and eigenmodes. The continuum spectrums are obtained by setting the determinant of the coefficients of the second-order derivative terms to zero. The eigenmodes are solved as a matrix eigenvalue problem $\mathbf{AX} = \lambda\mathbf{BX}$, with $\omega^2 = \lambda$ and $\mathbf{X} = [\cdots, \delta\phi_{m-1}, \delta\phi_m, \delta\phi_{m+1}, \cdots]^T$.

Eigenvalue solver (cont.)

Eq.(2) supports a wide range of modes such as Alfvén eigenmodes (GAE, TAE, RSAE and more) and unstable internal kink mode as well as ideal ballooning mode (IBM).

Zero boundary conditions. Central difference discrete: $\frac{df}{dr} = \frac{f_{j+1}-f_{j-1}}{2\Delta r}$ and $\frac{d^2f}{dr^2} = \frac{f_{j+1}-2f_j+f_{j-1}}{\Delta r^2}$.

The eigen matrix dimension is $(N_m \times N_r)^2$, where N_r is radial grid number and $N_m = m_{max} - m_{min} + 1$ is number of m mode numbers. Sparse matrix is used to speed up. Typical run time is **seconds** or less [Other codes usually minutes or more].

Cylinder global Aflvén eigenmode

$\rho = 1.0 - 0.98(r/a)^2$, $q = 1.001 + 2.0(r/a)^2$, $\beta = 0$, $n = 0$, $m = 2$ [PoP, 16, 072505].

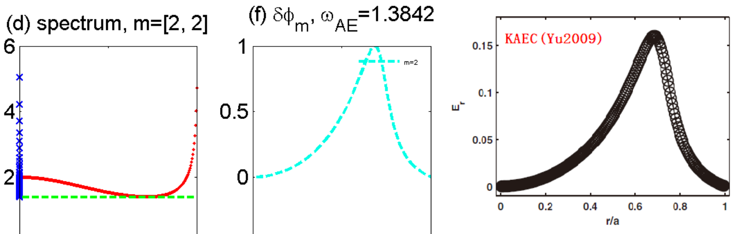


FIG. 1. The eigenfunction of GAEs: E_r vs r/a for the previous code (solid line), where the eigenvalue is $\omega^2/\omega_A^2(0)=1.916\,311\,630\,730\,91$; in this code (open circles), the eigenvalue is $\omega^2/\omega_A^2(0)=1.916\,311\,218\,715\,662$, where a is the minor radius and $\omega_A(0)$ is the frequency in the center of tokamaks.

Figure 1: GAE in AMC and KAEC codes, $\omega_{\mathrm{GAE}}^{\mathrm{AMC}} = 1.3842$ and $\omega_{\mathrm{GAE}}^{\mathrm{KAEC}} = 1.3843$.

Fu1989 TAE

$q = 1.0 + 1.0(r/a)^2$, $\rho = 1.0$, n=1 and $R_0/a = 4$ [PFB, 1, 1949].

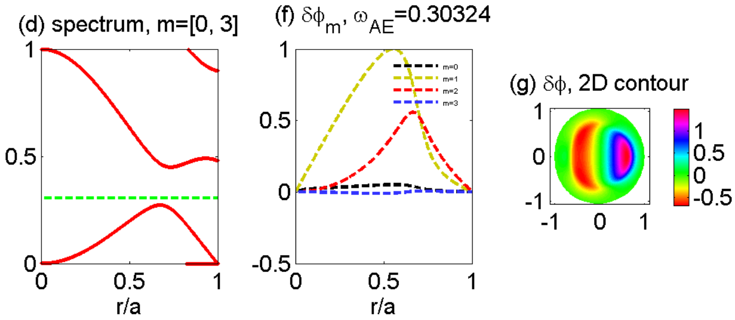


Figure 2: TAE in Fu1989, $\omega_{\rm TAE}^{\rm Fu89} = 0.31$, $\omega_{\rm TAE}^{\rm NOVA} = 0.3127$, $\omega_{\rm TAE}^{\rm KAEC} = 0.302$, $\omega_{\rm TAE}^{\rm AMC} = 0.303$.

Even and odd TAEs

$$q = 1.35 + 1.2(r/a)^2,\ \rho = 1/[1 + 2.0(r/a)^2],\ \text{n=1 and } R_0/a = 4$$

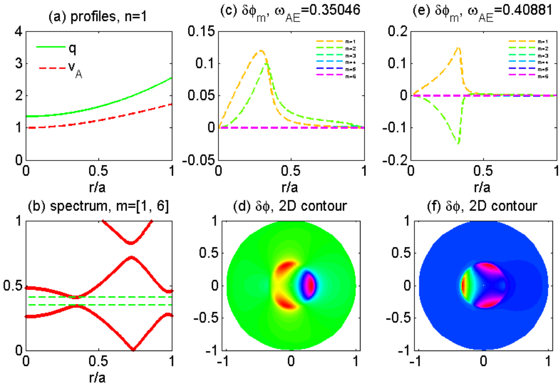


Figure 3: Odd and even TAEs. $\omega^{\mathrm{NOVA}}_{\mathrm{Odd}} = 0.4050$, $\omega^{\mathrm{KAEC}}_{\mathrm{Odd}} = 0.4086$, $\omega^{\mathrm{AMC}}_{\mathrm{Odd}} = 0.4088$; $\omega^{\mathrm{NOVA}}_{\mathrm{Even}} = 0.3550$, $\omega^{\mathrm{KAEC}}_{\mathrm{Even}} = 0.3523$, $\omega^{\mathrm{AMC}}_{\mathrm{Even}} = 0.3505$.

Reversed shear Alfvén eigenmodes

Deng2010 [PoP, 17, 112504]

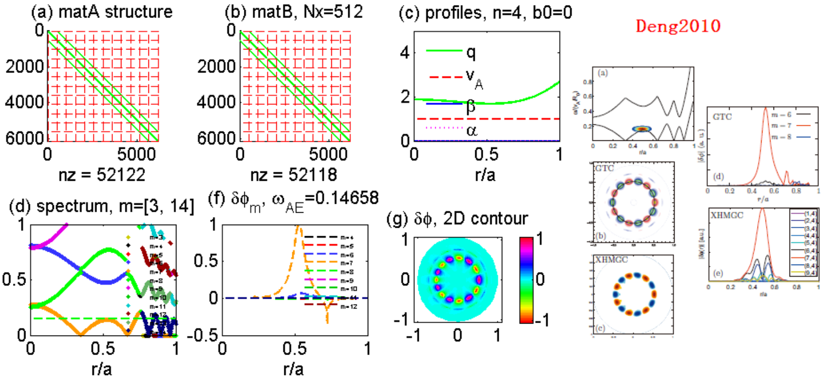


Figure 4: RSAE in Deng2010, $\omega_{\mathrm{RSAE}}^{\mathrm{GTC}} = 0.135$, $\omega_{\mathrm{RSAE}}^{\mathrm{HMGC}} = 0.160$, $\omega_{\mathrm{RSAE}}^{\mathrm{AMC}} = 0.147$, $\omega_{\mathrm{RSAE}}^{\mathrm{accum}} = 0.142$.

Ballooning mode

Test run

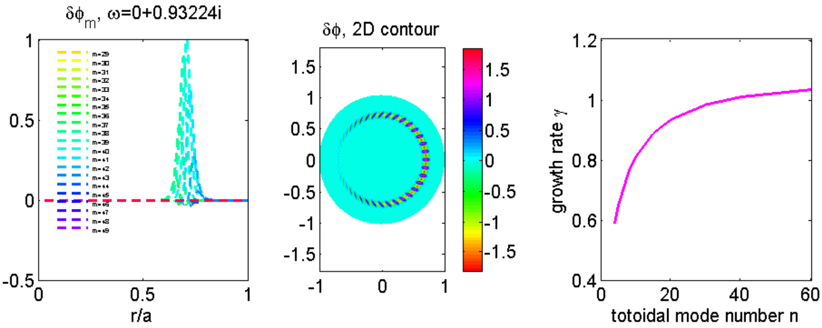
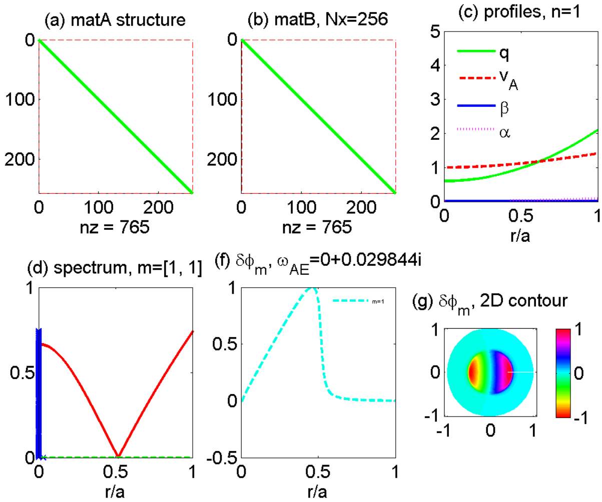


Figure 5: $\delta\phi_m(r)$, $\delta\phi(r,\theta)$ for $n = 20$ mode and γ v.s. n.

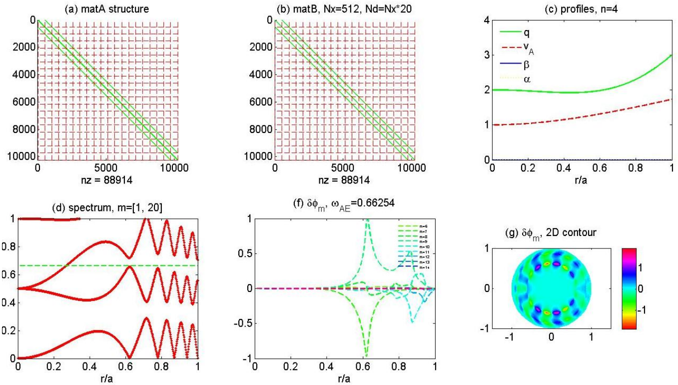
Internal kink

Test run



'EAE' and 'NAE'

High order $(m \pm 2, 3, \cdots)$ gap AEs



For 'EAE' and 'NAE', the non-circular geometry is not a must!

Sweeping RSAEs in HL-2A

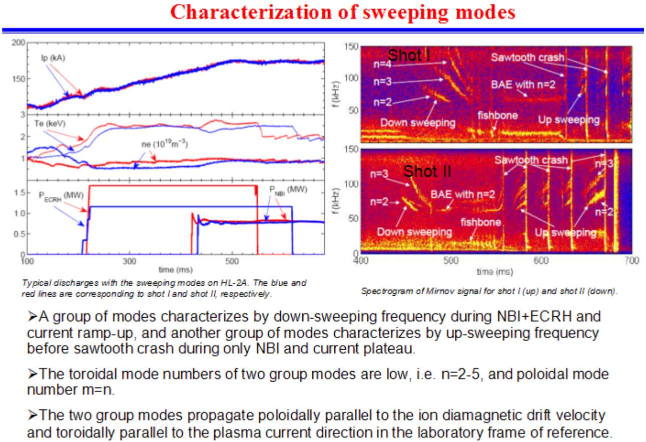


Figure 6: W. Chen *et al.*, 13th IAEA-TM EP, 17-20 September 2013, Beijing, China.

Sweeping RSAEs in HL-2A

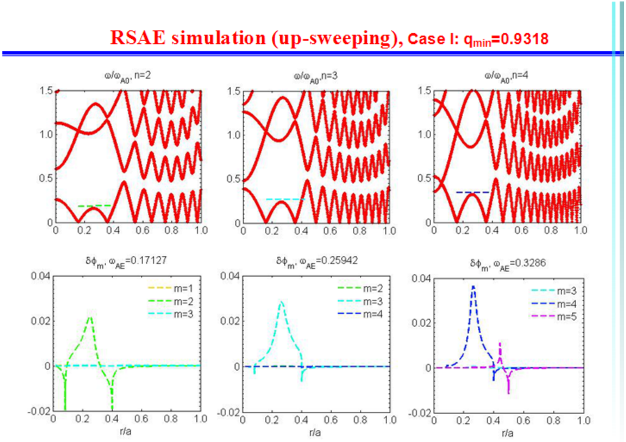
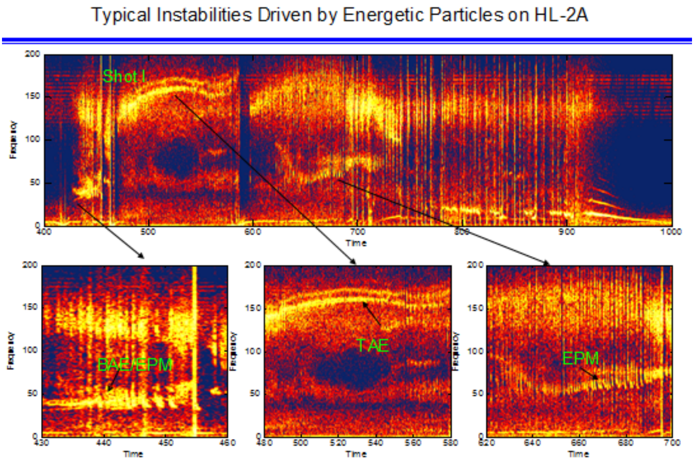


Figure 7: W. Chen *et al.*, 13th IAEA-TM EP, 17-20 September 2013, Beijing, China.

More data to be understood in HL-2A

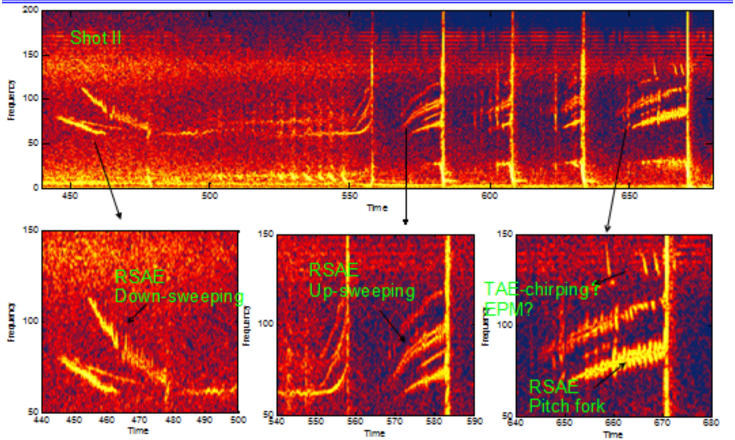


Need more research on experiment and theory, such as measurement and calculation of mode structures

From W. Chen

More data to be understood in HL-2A (cont.)

Typical Instabilities Driven by Energetic Particles on HL-2A (cont.)



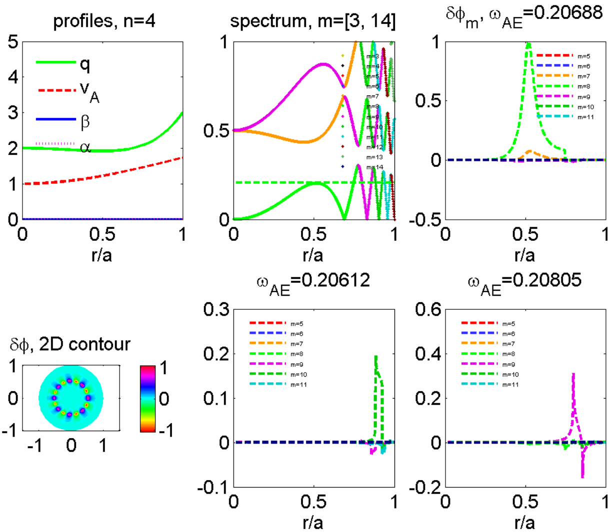
From W. Chen

Parallel equilibrium current effects on the existence of RSAE

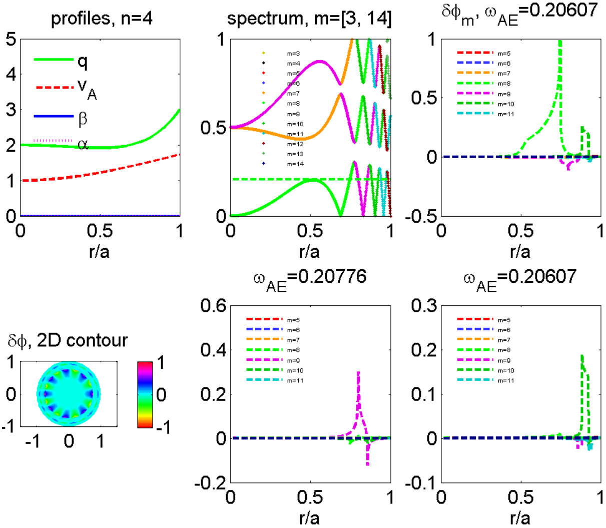
RSAEs existence criterion [Berk2001 PRL] $Q_{\mathrm{eff}} = Q_{\mathrm{f}} + Q_{\mathrm{tor}} + Q_{\mathrm{kink}} + Q_{\mathrm{pressure}} + Q_{\mathrm{kinetic}} + \ldots > Q_{\mathrm{critical}} = 1/4$

$J_{\|0}$ effects on RSAE

Without $J_{\|0}$



With $J_{\parallel 0}$ (No RSAE)



Summary

1. A fast and easily used global eigenvalue code is developed.

2. Good agreements with other codes.

3. Can be used as a tool for understanding the experiments and large scale simulations.

Future works

1. Extending it to more complicated models (e.g., adding kinetic effects).
2. Applying it for ballooning mode study, especially to benchmark GTC.

Tearing mode

Reduced MHD equation for cylinder tearing mode

$$\begin{cases} \partial_t \Psi = [\Psi, \phi] + \eta \nabla_\perp^2 \Psi + \partial_\varphi \phi, \\ \partial_t U = [U, \phi] + [\Psi, j_\varphi] + \partial_\varphi j_\varphi + \nu \nabla_\perp^2 U. \end{cases} \tag{5}$$

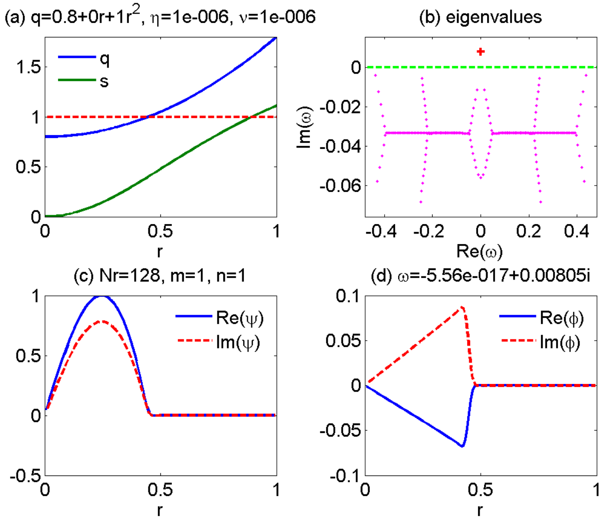
$U = \nabla_\perp^2 \phi$, $j_\varphi = \nabla_\perp^2 \Psi$.

$$\begin{cases} [f, g] = \frac{1}{r}\left(\frac{\partial f}{\partial r}\frac{\partial g}{\partial \theta} - \frac{\partial g}{\partial r}\frac{\partial f}{\partial \theta}\right) = \frac{im}{r}\left(g\frac{\partial f}{\partial r} - f\frac{\partial g}{\partial r}\right), \\ \nabla_\perp^2 = \frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} = \frac{1}{r}\left(\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\right) - \frac{m^2}{r^2}. \end{cases} \tag{6}$$

Equilibrium relations: $q^{-1} = -\frac{1}{r}\frac{d}{dr}\Psi_0$, $j_0 = \nabla_\perp^2 \Psi_0 = -\frac{1}{r}\frac{d}{dr}\frac{r^2}{q}$, $s = \frac{r}{q}\frac{dq}{dr}$, $U_0 = \phi_0 = 0$.

Similar treatment will be used to extend AMC for toroidial tearing mode.

Tearing mode



Double tearing mode