Parallel Equilibrium Current Effect on Existence of Reversed Shear Alfvén Eigenmodes

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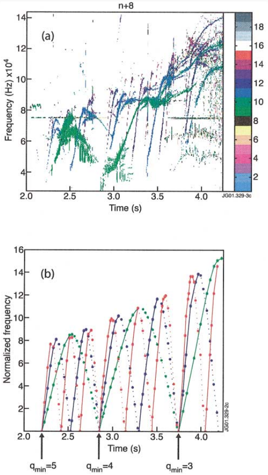
Motivation

1. Inspired by HL-2A recent experiment [Chen *et al*, 13]. NOVA [Cheng86] can not find a well mode structure RSAE. KAEC [Yu09] can find a similar eigenmode as found in experiment when including kinetic effects or excluding kink term. **WHY?**
2. New fast global eigenvalue code **AMC (Alfvén Mode Code)** for large scale simulations (e.g., GTC) & experiments (e.g., HL-2A, J-TEXT).
3. Improve several inaccurate (model equation) expressions in literatures.

Reversed shear Alfvén eigenmodes

1. RSAE (or Alfvén cascade modes), localized around $q_{\rm min}$, reversed shear profile.
2. Frequency sweeps up/down when $q_{\rm min}$ drops. $\omega_{\rm RSAE} \simeq \frac{v_A}{R}\left|\frac{m}{q_{\rm min}(t)} - n\right|$.
3. Experiments: Kimura98, Sharapov01, Nazikian03, [HL-2A, Chen *et al*, 13] ...
4. Theoretically, existence of RSAEs well studied: energetic particle [Berk01], toroidicity [Breizman03], pressure / pressure gradient [Breizman05,Fu06,Yu13], kinetic [Yu09], ...
5. **Limitations of previous studies of parallel equilibrium current (kink) term: Qualitative.** [Deng10&12].



Berk *et al*, 2001, PRL.

Starting equation

Vorticity equation (shear Alfvén law)

$$\underbrace{\nabla\cdot\left(\frac{\omega^2}{v_A^2}\nabla_\perp\delta\phi\right)}_{\text{inertial}}+\underbrace{\mathbf{B}\cdot\nabla\left(\frac{1}{B^2}\nabla\cdot B^2\nabla_\perp Q\right)}_{\text{field line bending}}-\underbrace{\nabla\left(\frac{J_\|}{B}\right)\cdot(\nabla Q\times\mathbf{B})}_{\text{kink / parallel equilibrium current}}+\underbrace{2\frac{\kappa\cdot(\mathbf{B}\times\nabla\delta P)}{B^2}}_{\text{ballooning}}=0, \quad (1)$$

$\kappa=\mathbf{b}\cdot\nabla\mathbf{b}$, $Q=(\mathbf{b}\cdot\nabla\delta\phi)/B$, $\delta P=(\mathbf{b}\times\nabla\delta\phi\cdot\nabla P)/B$, $J_\|=\mathbf{b}\cdot\nabla\times\mathbf{B}$.

Shifted circular geometry. Second order for $\epsilon=r/R\ll 1$, $\beta\sim O(\epsilon^2)$.

Feature 1 (the equation): Terms separated well, good for theoretical study.
NOVA: 1. numerical equilibrium; 2. solves original MHD equation. Difficult to separate different effects.

We solve below coupled equation

$\delta\phi = \sum \delta\phi_m(r)\exp(in\zeta - im\theta)$, expanding Eq.(1) to $O(\epsilon^2)$, to a coupled equation

$$\mathbf{L_{m,m-1}}\delta\phi_{\mathbf{m-1}} + \mathbf{L_{m,m}}\delta\phi_{\mathbf{m}} + \mathbf{L_{m,m+1}}\delta\phi_{\mathbf{m+1}} = \mathbf{0}, \tag{2}$$

$$L_{m,m} = \frac{\partial}{\partial r}\Big[\frac{(1+4\epsilon\Delta')}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2\Big] r\frac{\partial}{\partial r} + (k_m^2)' - \frac{m^2}{r}\Big\{\frac{[1-4\epsilon(\epsilon+\Delta')]}{v_A^2}\bar{\omega}^2 - k_m^2 - c_s^2 - \bar{\kappa}_r\alpha/q^2\Big\}, \tag{3}$$

$$L_{m,m\pm1} = \bar{\omega}^2\Big\{\frac{\partial}{\partial r}\frac{(2\epsilon+\Delta')}{v_A^2} r\frac{\partial}{\partial r} - \frac{(\epsilon-\Delta')}{v_A^2}\frac{m(m\pm1)}{r} \mp \frac{[\epsilon+(r\Delta')']}{v_A^2} m\frac{\partial}{\partial r}\Big\} - \Big\{\frac{\partial}{\partial r} r\Delta' k_m k_{m\pm1}\frac{\partial}{\partial r} - \frac{m^2}{r}(\epsilon+\Delta')k_m k_{m\pm1} \mp m[\epsilon+(r\Delta')']k_m k_{m\pm1}\frac{\partial}{\partial r}\Big\} - \frac{m\alpha}{2q^2}\Big(\frac{m}{r} \mp \frac{\partial}{\partial r}\Big). \tag{4}$$

$\bar{\omega} = \omega/(V_A/R_0)$, $V_A = \langle v_A(r,\theta)\rangle$, $k_m = (n - m/q)$.

Eigenvalue solver

1. Different authors may give different forms of $L_{m,m}$ and $L_{m,m\pm1}$. Some (Fu06, Breizman05, Vlad99, $\cdots$) may break the **self-adjointness**[1]. Ours are self-adjoint (all eigenvalues ω^2 are real).
2. Continuum spectrums: setting the determinant of the coefficients of the second-order derivative terms to zero.
3. Eigenmodes: $\mathbf{AX}=\lambda\mathbf{BX}$, $\omega^2=\lambda$, $\mathbf{X}=[\cdots,\delta\phi_{m-1},\delta\phi_m,\delta\phi_{m+1},\cdots]^T$. Zero boundary condition.
4. **Feature 2 (the code): Supports AEs (GAE, TAE, RSAE and more), unstable kink & ballooning.** More extensions (tearing, kinetic, EPM, flow, ...) on the way.
5. Eigen matrix dimension $(N_m\times N_r)^2$, $N_m=m_{max}-m_{min}+1$. Sparse matrix and standard eigenvalue solver (e.g., *eigs* in MATLAB) to speed up.
6. **Feature 3 (the code): Fast and easily used significantly.** Typical run time: **seconds** or less. Other codes (NOVA, KAEC, GTAW, ...): minutes or more.

[1] Non-self-adjointness will give non-physical solutions. Details in [Xie2015PoP].

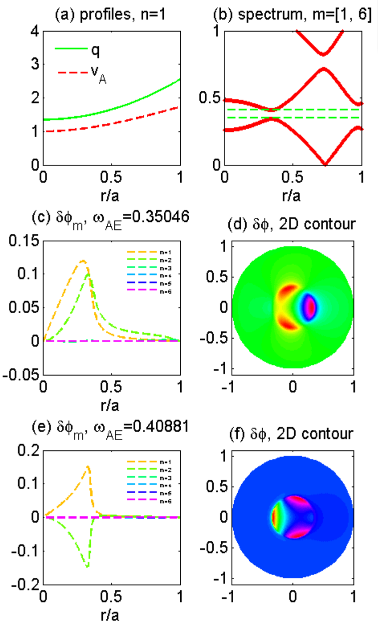
Benchmark

Agreed well with NOVA, KAEC, GTC and HMGC for GAE, TAE, RSAE.

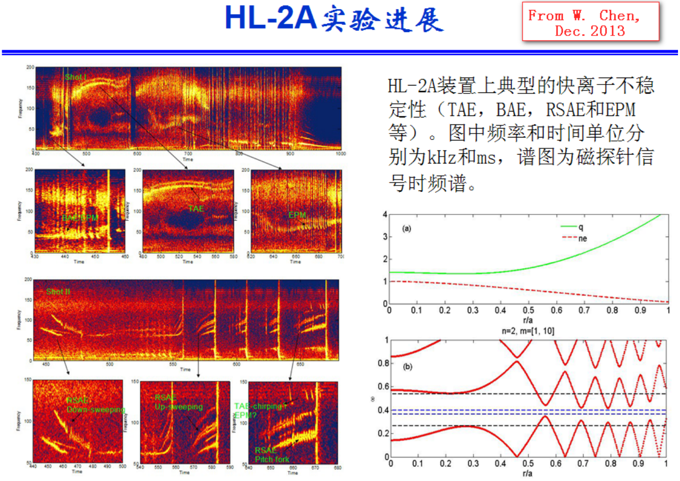
Typical benchmark: Odd and even TAEs.

$q = 1.35 + 1.2(r/a)^2$,
$\rho = 1/[1 + 2.0(r/a)^2]$, n=1,
$R_0/a = 4$.

-	NOVA	KAEC	AMC
ω_{Odd}	0.4050	0.4086	0.4088
ω_{Even}	0.3550	0.3523	0.3505



Successfully applied to the HL-2A experiments



AMC frequencies and RSAE sweeping agree with experiment.

RSAEs existence criterion[2] (theory)

1. Assume single m dominant, dimensionless equation for RSAE

$$\frac{\partial}{\partial x}(S+x^2)\frac{\partial}{\partial x}\delta\phi_m+(Q-S-x^2)\delta\phi_m=0, \tag{5}$$

   $x=m(r-r_0)/r_0$, r_0 the radius of $q_{\rm min}$.

2. RSAEs existence criterion
   $Q_{\rm eff}=Q_{\rm f}+Q_{\rm tor}+Q_{\rm pressure}+Q_{\rm kinetic}+...>Q_{\rm critical}=1/4$. These terms can be either favorable or unfavorable. $Q_{\rm eff}$ as **Schrödinger potential**, $Q_{\rm critical}$ similarly as Suydam's criterion.

3. The above analytical calculations are not rigorous.

[2] Berk *et al*, 2001, PRL.

Q terms (theory)

$Q_{\rm tor}$ usually small: the pure toroidicity factor difficult to make RSAE exist.

$$Q_{\rm tor} = 2\frac{mq_0^2(-k_{m0})}{r_0^2 q_0''}\frac{(\epsilon^2+2\Delta'\epsilon)}{1-4k_{m0}^2 q_0^2}. \tag{6}$$

Our quantitative result:

Without kink term, $L_{m,m}^{\rm new} = L_{m,m} + 3k_m k_m' + rk_m k_m''$,

$$\mathbf{Q}_{\rm new} \simeq \frac{\mathbf{r_0 k_{m0}(k_m'')_0}}{\mathbf{r_0(k_m^2)_0''/2}} \simeq \mathbf{1}. \tag{7}$$

always larger than zero (with kink, $Q_{\rm new}=0$), also easy larger than $Q_{\rm critical} = 1/4$

$\Rightarrow$ **parallel equilibrium current always (strongly) unfavorable!**

Case 1 (numerical)

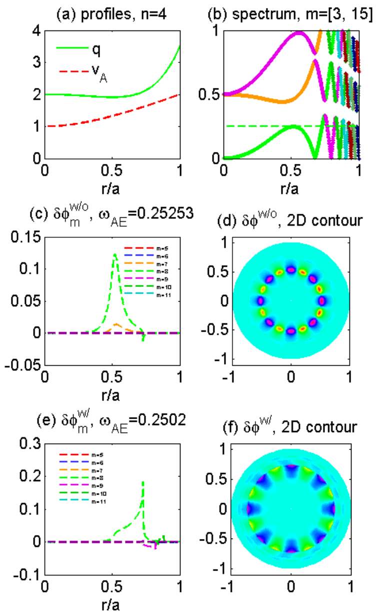


Figure: Good global RSAE mode only exist when kink term is removed.

$$q(r) = q_m + c_1(r^2 - r_m^2)^2 + c_2(r^2 - r_m^2)^3,$$

$$v_A^2(r) = 1/(1+3r^2).\ n = 4,\ R_0/a = 5,$$

$$q_m = 1.91,\ q_0 = 2.0,\ q_a = 3.5,\ r_m = 0.5.$$

$$Q_{\rm tor} = 0.2578,\ Q_{\rm new} = 0.5184.$$

Case 2 (numerical)

Can pure toroidicity factor ($Q_{\rm f}=Q_{\rm pressure}=Q_{...}=0$ but $Q_{\rm tor}\neq 0$) make RSAE exist in global calculations?

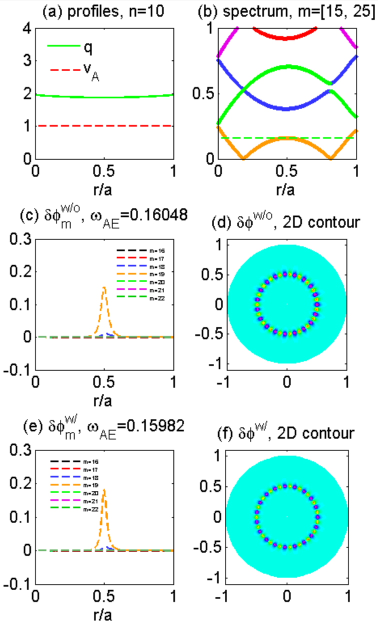
Yes, although difficult!

$q(r)=\frac{q_0}{[1-(x-0.5)^2/w_q^2]}$, $v_A^2(r)=1$.

To make $Q_{\rm tor}\gg 1/4$.

Figure: Good global RSAE mode exist for both with and w/o kink term cases.

$n=10$, $R_0/a=5$, $q_m=1.87$, $w_q=2.5$.

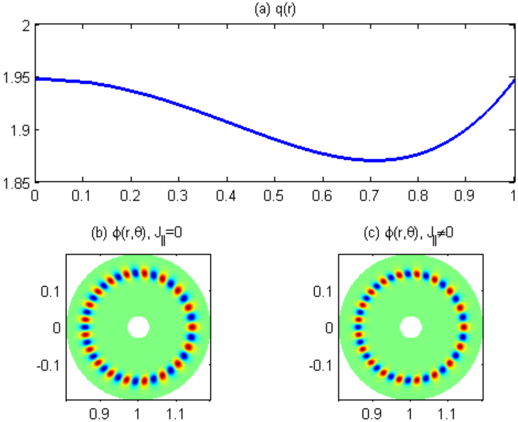


GTC verification (simulation)

RSAE exists for both with and w/o kink term cases.

$v_A^2(r) = 1.$

$n = 10,\ R_0/a = 5.0,$
$q_{\rm min} = 1.87.$



GTC simulation of RSAE: (a) q(r) profile; (b & c) ϕ on poloidal plane w/o and with kink term.

Discussions Case 2 & GTC case

1. For Case 2 and GTC case, the mode structures for both with and w/o $J_{\|0}$ are similar though a slight difference in frequency.

2. Indicates that kink term mainly affects whether RSAE can exist, but affects little for the mode structure when RSAE has existed.

3. Since the effect of each terms in $Q_{\rm eff}$ are just a summation, **for simplicity** in equations and simulations, we can use this $Q_{\rm new}$ to replace other terms. That is, to make RSAE exist, **we can suppress kink term artificially instead of adding fast particles, pressure and so on**.

4. However, **this suggestion is only useful for numerical studies**, since that all effects should exist in experiments.

Summary[3]

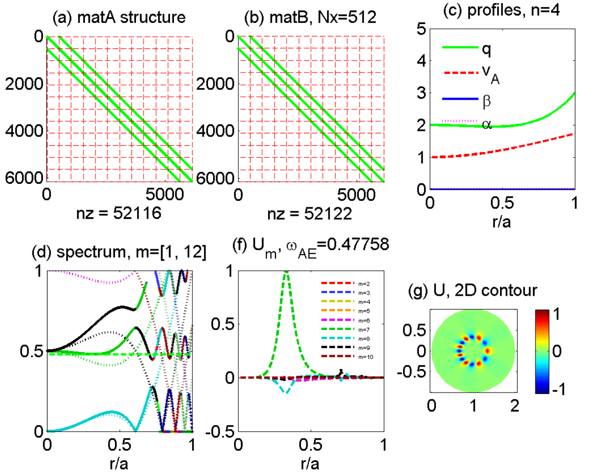
1. Clarified that the equilibrium parallel current $Q_{\rm new}$ term is always **(strongly) unfavorable**, and the artificial suppression of this term in equations or simulations will help to find RSAEs.

2. At ideal MHD and zero-pressure limits, the main possible favorable term is the toroidicity term $Q_{\rm tor}$. Though usually small, the toroidicity effect can also make RSAE exist under same parameters.

3. **Other contributions** of this work: several inaccurate expressions in literatures have been improved and a new fast and easily used global eigenvalue code is constructed, for studying the Alfvén modes in tokamak plasma.

[3] [Xie2015] H. S. Xie & Y. Xiao, Phys. Plasmas, 22, 022518 (2015). AMC and awcon codes: http://ifts.zju.edu.cn/student/hsxie/codes/amc/

Down-sweeping RSAE



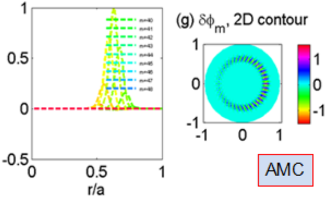
Down-sweeping RSAE was also found in AMC model. The existence of this interesting mode very sensitive to parameters.

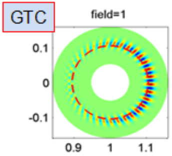
IBM

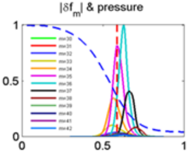
2-2. Ideal Ballooning (1/3)

w/o shift

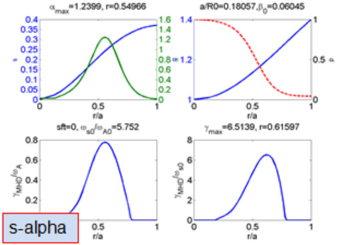
	gamma	r/a (position)
s-alpha, n-> infty	6.51	0.62
gtc, n=30	6.7	0.60
amc-reduce, n=30	5.75	0.63





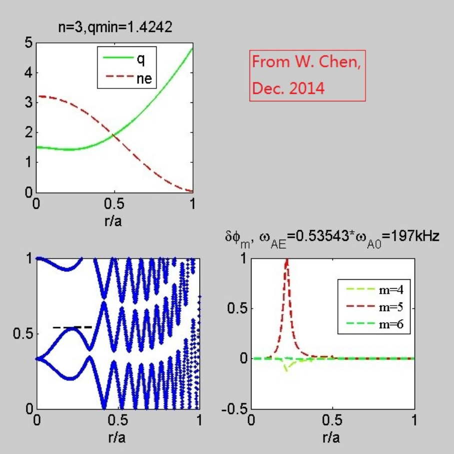


well agree



IBM benchmark: AMC, local s-α, GTC.

Gap AEs



Gap AEs agree with HL-2A recent experiments.