Simple PDRK User Guide

-A powerful new kinetic plasma dispersion relation solver

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Introduction

- Why kinetic dispersion relation important? – Waves and instabilities are one of the most important feature of plasma.

- Why difficult to solve? –Many branches, difficult to convergent in some parameters.

- What is PDRK?– The first kinetic plasma dispersion relation solver that can give all the important solutions at one time without requiring initial guess for root finding.

Equations

PDRK v181027 solves uniform plasma dispersion relation with loss-cone drift bi-Maxwellian equilibrium distribution function.

Using Matlab, via matrix transformation method. (Also python version provided by Dr. Xin TAO at USTC)

For D(omega, k)=0, give k, solve series omega(s).

$$f_{s0}(v_\parallel, v_\perp) = f_\perp(v_\perp) f_z(v_\parallel) \tag{1}$$

$$= \frac{1}{\pi^{3/2} v_{zts} v_{\perp ts}^2} \exp\left[-\frac{(v_\parallel - v_{ds})^2}{v_{zts}^2}\right]\left\{\Delta_s \exp\left(-\frac{v_\perp^2}{v_{\perp ts}^2}\right) + \frac{1-\Delta_s}{1-\alpha_s}\left[\exp\left(-\frac{v_\perp^2}{v_{\perp ts}^2}\right) - \exp\left(-\frac{v_\perp^2}{\alpha_s v_{\perp ts}^2}\right)\right]\right\},$$

The dispersion relation is

$$|\boldsymbol{D}(\omega, \boldsymbol{k})| = \begin{vmatrix} K_{xx} - \frac{c^2k^2}{\omega^2}\cos^2\theta & K_{xy} & K_{xz} + \frac{c^2k^2}{\omega^2}\sin\theta\cos\theta \\ K_{yx} & K_{yy} - \frac{c^2k^2}{\omega^2} & K_{yz} \\ K_{zx} + \frac{c^2k^2}{\omega^2}\sin\theta\cos\theta & K_{zy} & K_{zz} - \frac{c^2k^2}{\omega^2}\sin^2\theta \end{vmatrix} = 0, \tag{10}$$

The standard linearized kinetic theory gives [Ichimaru1973, p51]

$$\boldsymbol{K}(\omega, \boldsymbol{k}) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)\boldsymbol{I} + \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{\infty} \int d\boldsymbol{v} \frac{\boldsymbol{\Pi}_s}{\omega - k_\parallel v_\parallel - n\Omega_s}\left(\frac{n\Omega_s}{v_\perp}\frac{\partial f_{s0}}{\partial v_\perp} + k_\parallel \frac{\partial f_{s0}}{\partial v_\parallel}\right), \tag{11}$$

and

$$\boldsymbol{\Pi}_s = \begin{pmatrix} (\frac{n\Omega_s}{k_\perp}J_n)^2 & i\frac{n\Omega_s}{k_\perp}v_\perp J_n J_n' & \frac{n\Omega_s}{k_\perp}v_\parallel J_n^2 \\ -i\frac{n\Omega_s}{k_\perp}v_\perp J_n J_n' & (v_\perp J_n')^2 & -iv_\perp v_\parallel J_n J_n' \\ \frac{n\Omega_s}{k_\perp}v_\parallel J_n^2 & iv_\perp v_\parallel J_n J_n' & (v_\parallel J_n)^2 \end{pmatrix}, \tag{12}$$

where $\int d\boldsymbol{v} \equiv 2\pi\int_0^\infty v_\perp dv_\perp \int_{-\infty}^{\infty} dv_\parallel$, Bessel function $J_n = J_n(\frac{k_\perp v_\perp}{\Omega_s})$ and $\omega_p^2 = \sum_s \omega_{ps}^2$.

Solvers compare

	Initial guess?	Fast?	Support high harmonic mode?	Separate modes?	All solutions?
WHAMP [Ronnmark1982]	Must	Fast	Difficult	Difficult	No
NHD [Verscharen 2018]	Must	Middle	?	?	No
HOTRAY [Horne1989]	Must	Middle	Difficult	Difficult	No
PDRK [Xie2016]	**Not required**	Middle	**Easy**	**Easy**	**Yes**

What makes PDRK attractive? It solves the difficulty of root finding, i.e., not requires initial guess and can give all the important solutions at one time. You **do not need luck** any more.

At v181027, PDRK have support loss cone drift bi-Maxwellian distribution, for both electromagnetic and electrostatic cases.

Steps to run PDRK

Note, v181027 has combined all three models (em3d, es3d, es1d) in PDRK Xie2016PST original paper to a single version.

- ✓ EM3D: iem=1;
- ✓ ES3D: iem=0;
- ✓ ES1D: iem=0, theta=0.

Thus, it is extremely simple to switch between em run and es rum.

- 1. Set species parameters in 'pdrk.in';
- 2. Set 'setup.m', B0, k, theta, etc;
- 3. Run 'main.m';
- 4. After run 'plot_all.m', zoom in and select which branch(es) to further plot;
- 5. Set the 'wpdat' in 'pdrk_wpdat.m', 'plot_select.m' will search the solutions in the same branches in 'wpdat', and then store and plot them;
- 6. If you require polarization info, run 'output.m'

Typical cases 1: Cold plasma

pdrk.in

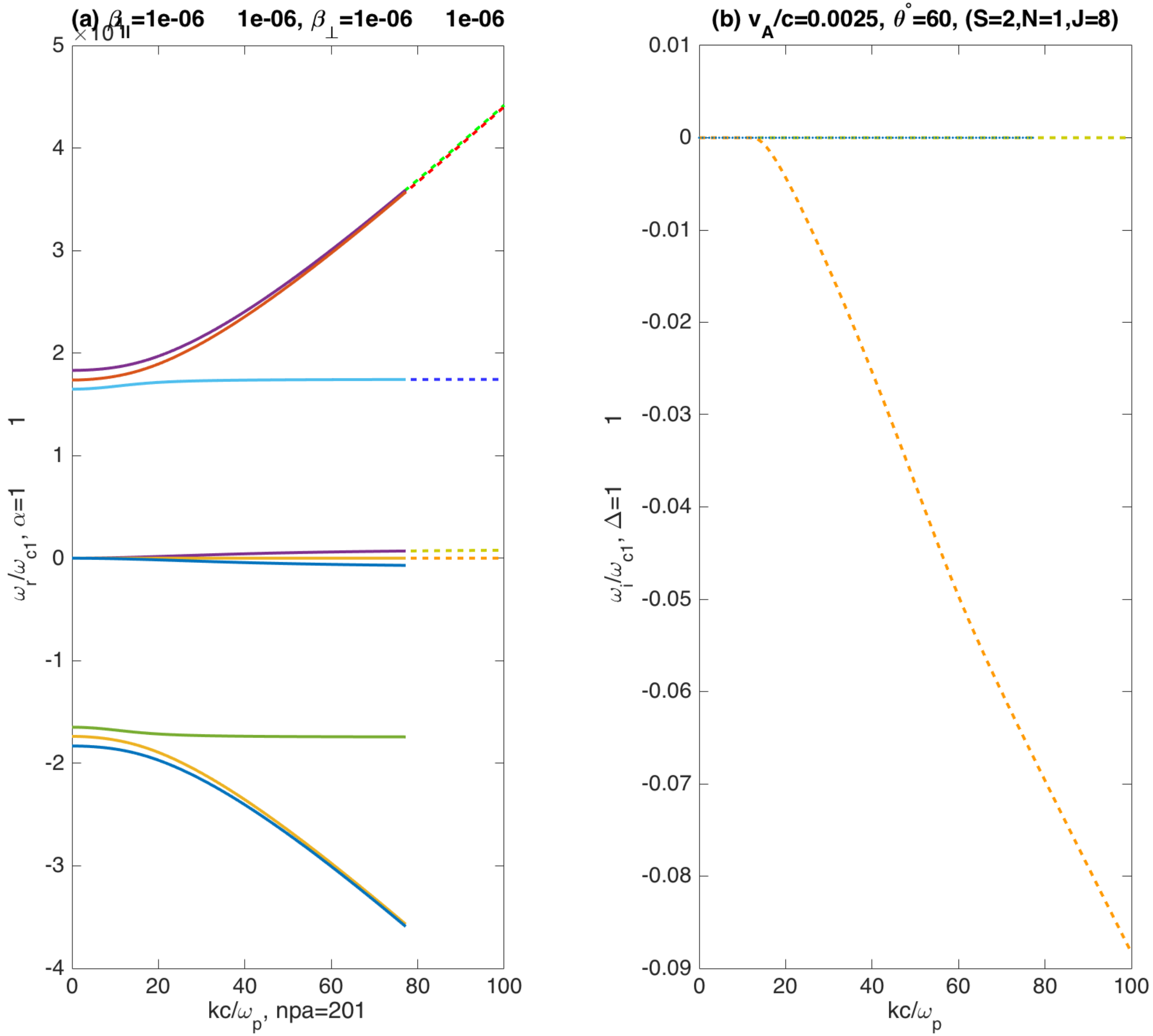
qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
1	1	8.7e6	2.857e-3	2.857e-3	1.0	1.0	0.0
-1	5.447e-4	8.7e6	2.857e-3	2.857e-3	1.0	1.0	0.0

```
B0=100.0E-9; N=1; J=8; iem=1;

(ipa,ipb) = (1,1) scan k, fixed theta=60

pa=0:0.5:100
```

Solid lines: Fluid solver PDRF results.

Dash lines: PDRK results.

We find good agreement, except a slight difference at large k for the ion cyclotron wave, which is damped due to kinetic effect.



Typical cases 2: Loss cone mirror

pdrk.in

qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
1	1	1.e6	24840.	49680.	0.5	0.1	0.0
-1	5.447e-4	1.e6	24840.	24840.	0.5	0.1	0.0

B0=100.0E-9; **N=3;** J=8; iem=1;

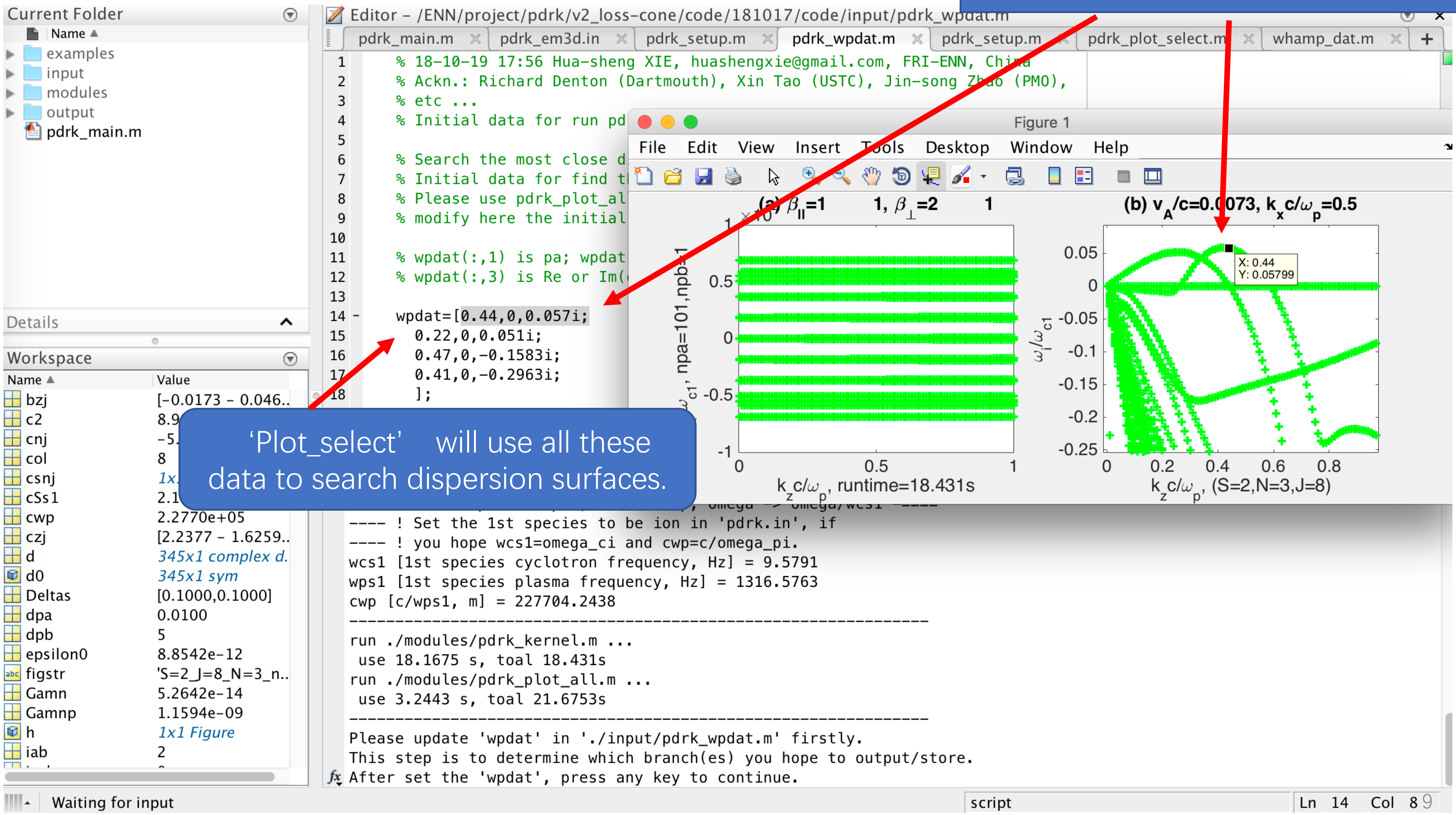
(ipa,ipb) = (3,3) scan kz, fixed kx=0.5

pa=0:0.01:1

Ensure your results are convergent by trying a larger N.

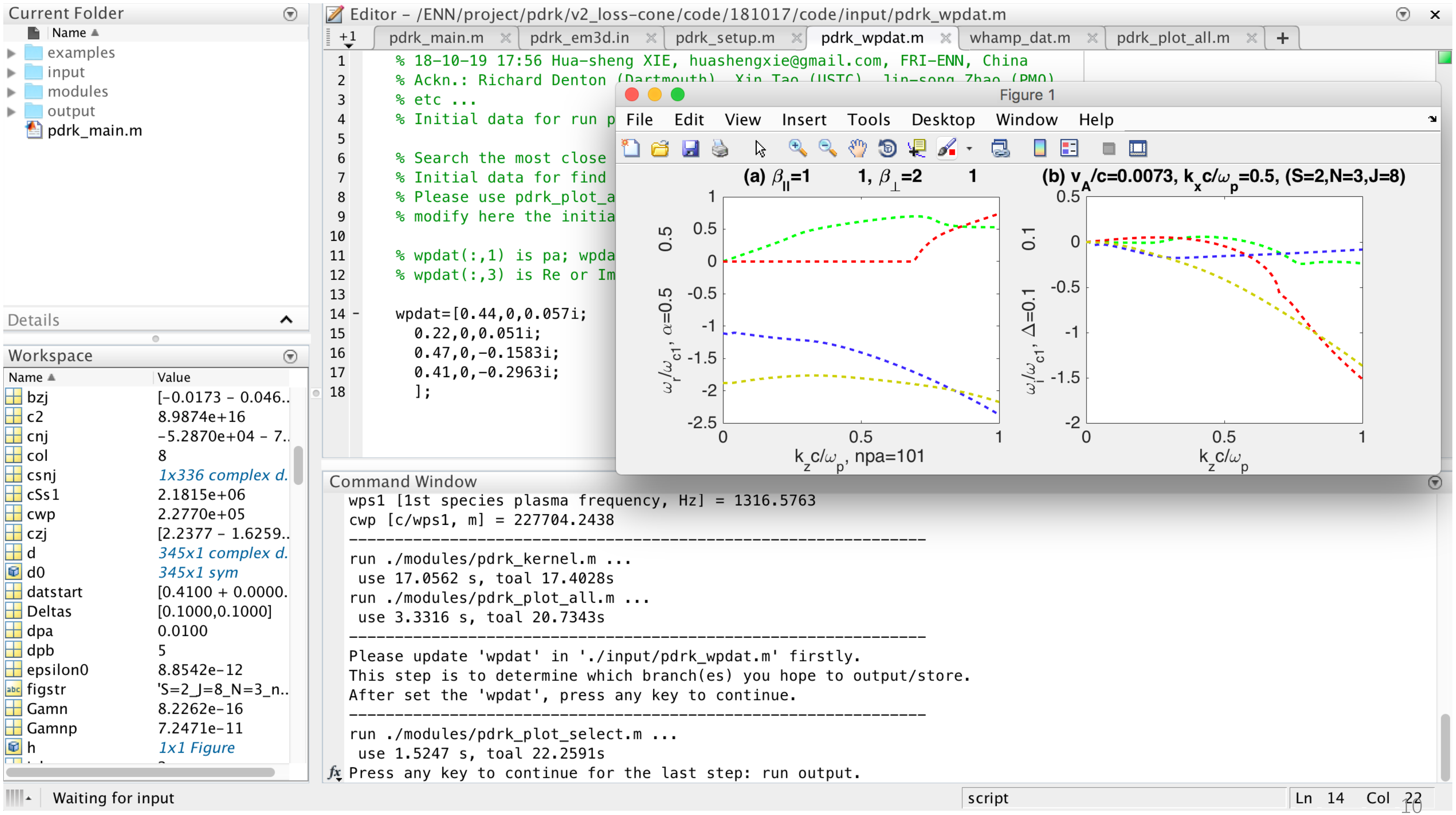
After run main.m and plot_all.m, zoom in the Fig(b) to select ‘wpdat’ .

Find a point in ‘plot_all’ figure and set it into ‘pdrk_wpdat.m’

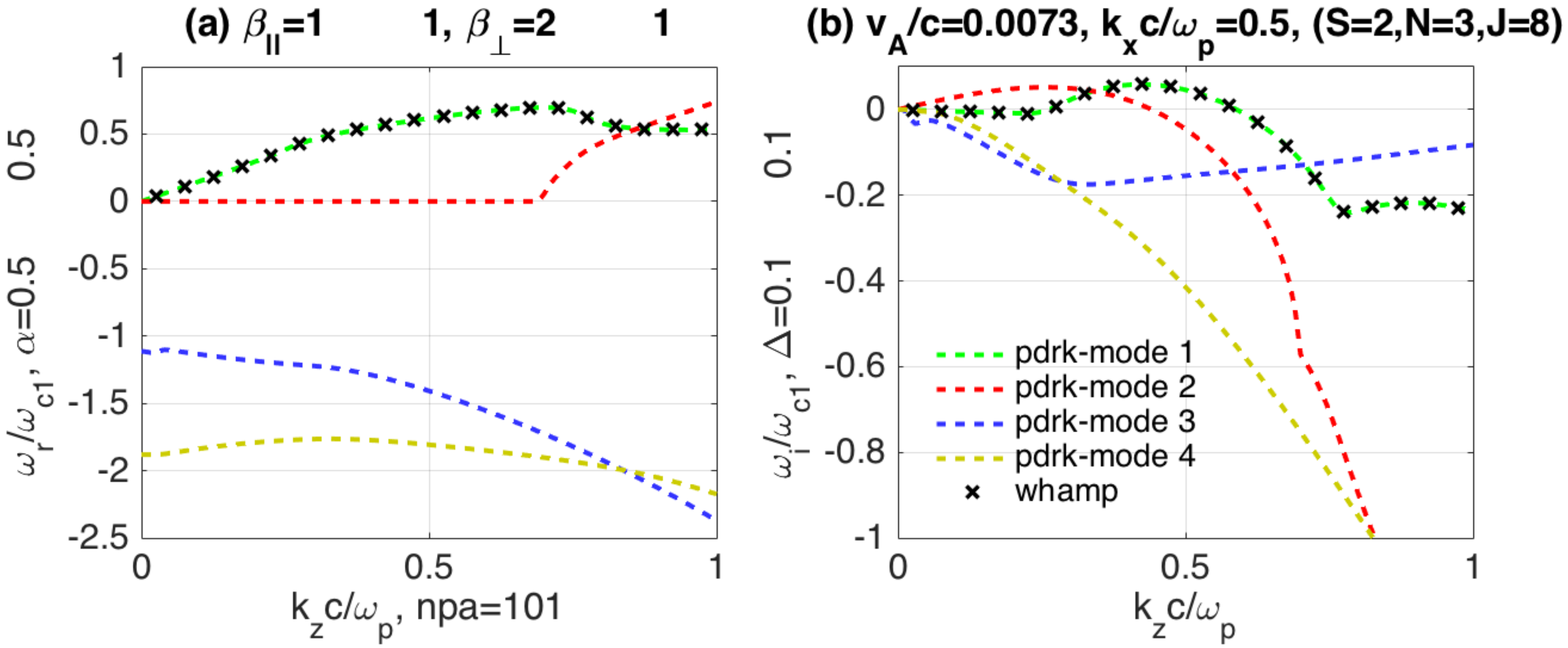


‘Plot_select’ will use all these data to search dispersion surfaces.

After set 'wpdat', press enter, we obtain the follow figure.



Compare to WHAMP data, we find good agreement. However, WHAMP can only find one solution at one time and require good initial guess for root finding.



Typical cases 3: Parallel Multi-species Beam mode

pdrk.in

qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
1	1	2.528e5	3.5387e4	3.5387e4	1.0	1.0	0.0
-1	5.447e-4	3.16e5	2.831e4	2.831e4	1.0	1.0	3.7013e-3
1	1	3.16e4	28.31e4	28.31e4	1.0	1.0	3.7013e-2
1	2	3.16e4	28.31e4	28.31e4	1.0	1.0	0.0

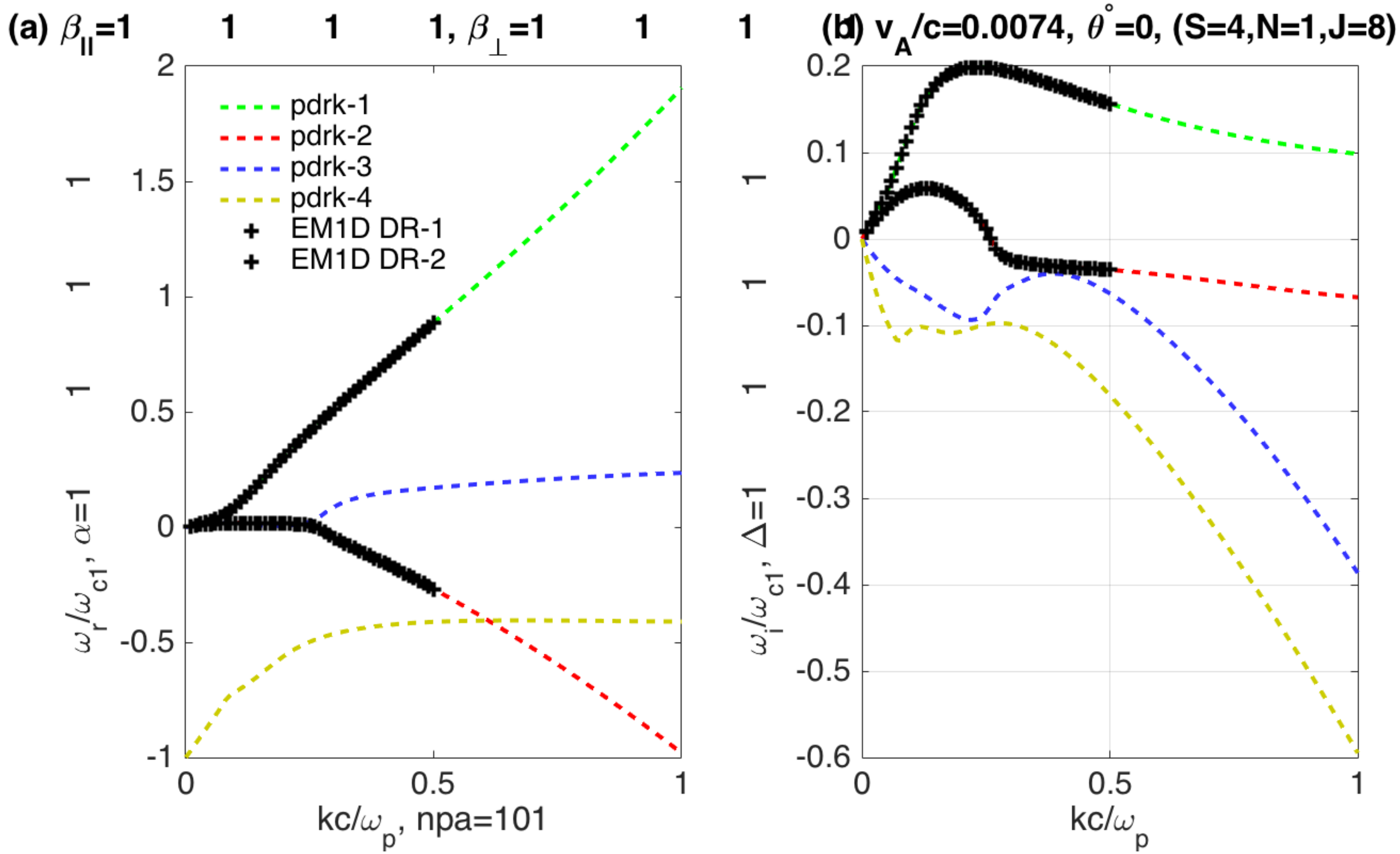
B0=60.0E-9; N=1; J=8; iem=1;

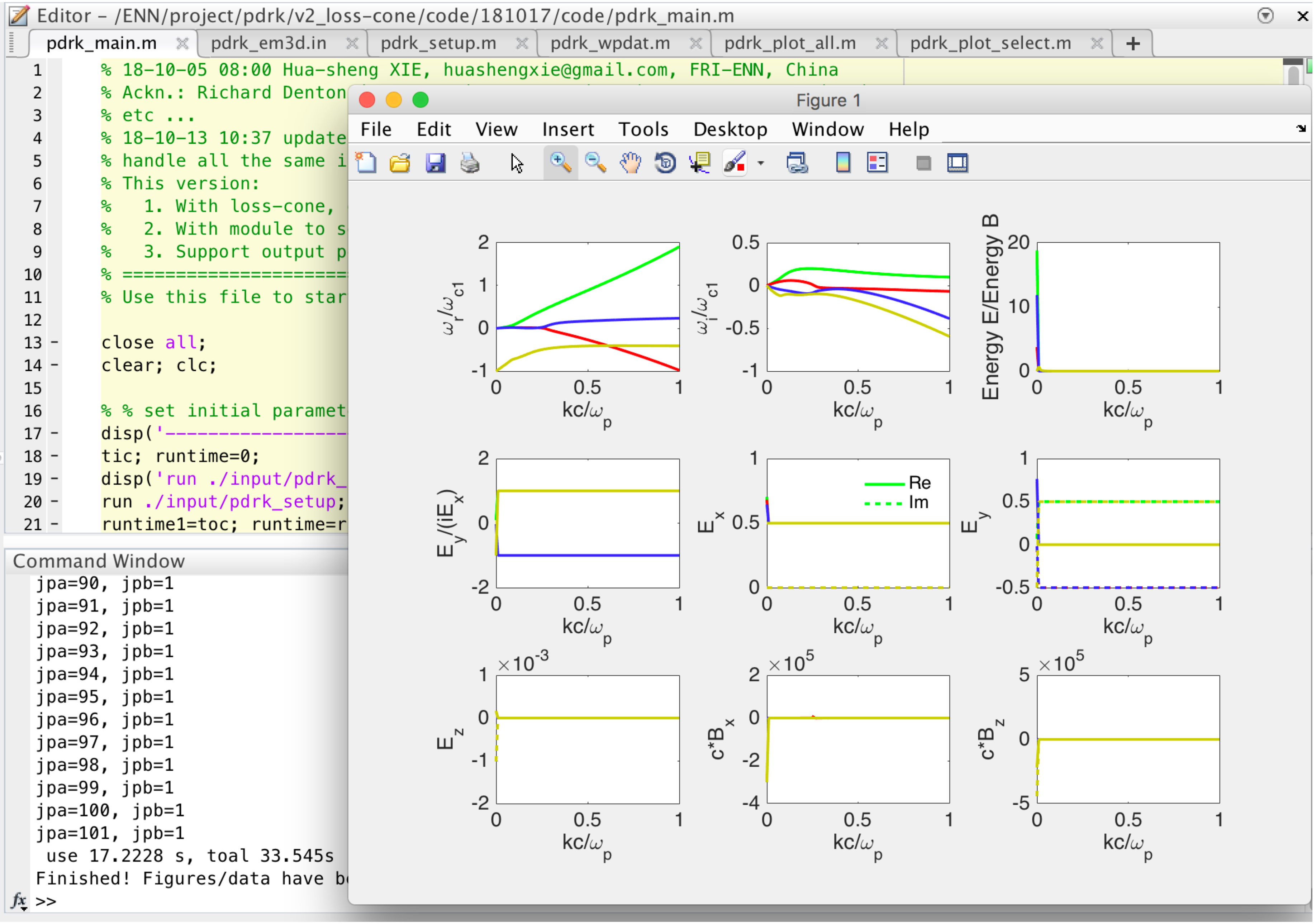
(ipa,ipb) = (1,1) scan k, fixed theta=0

pa=0:0.01:1;

iout=2;

Good agreement to EM1D theta=0 dispersion relation solutions.





Iout=2, polarization are also calculate.

We find Ey/(iE_x)=+1 or -1, i.e., only left and right-hand polarized modes. Agree with theory.

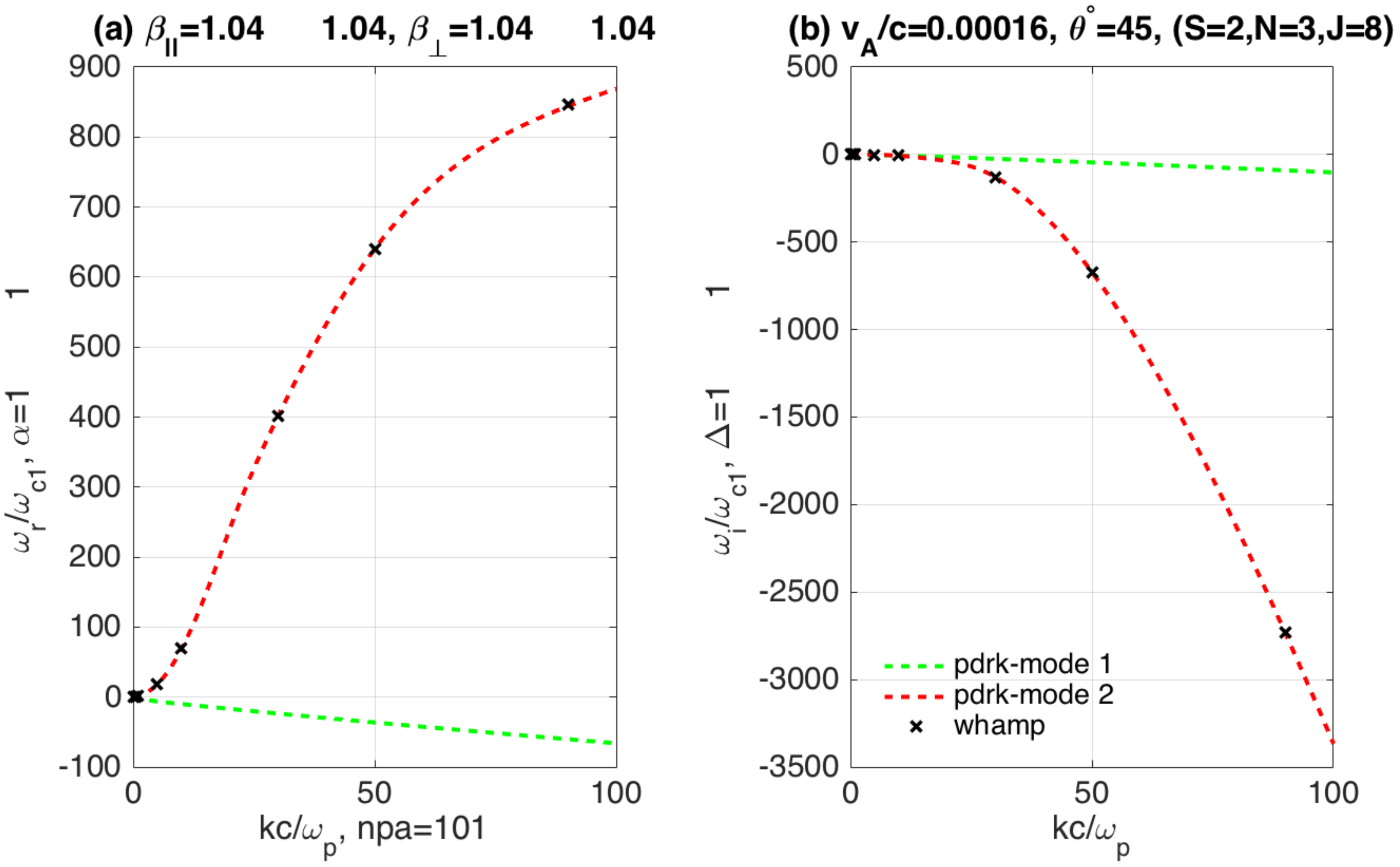
Typical cases 4: Dispersion surface

pdrk.in

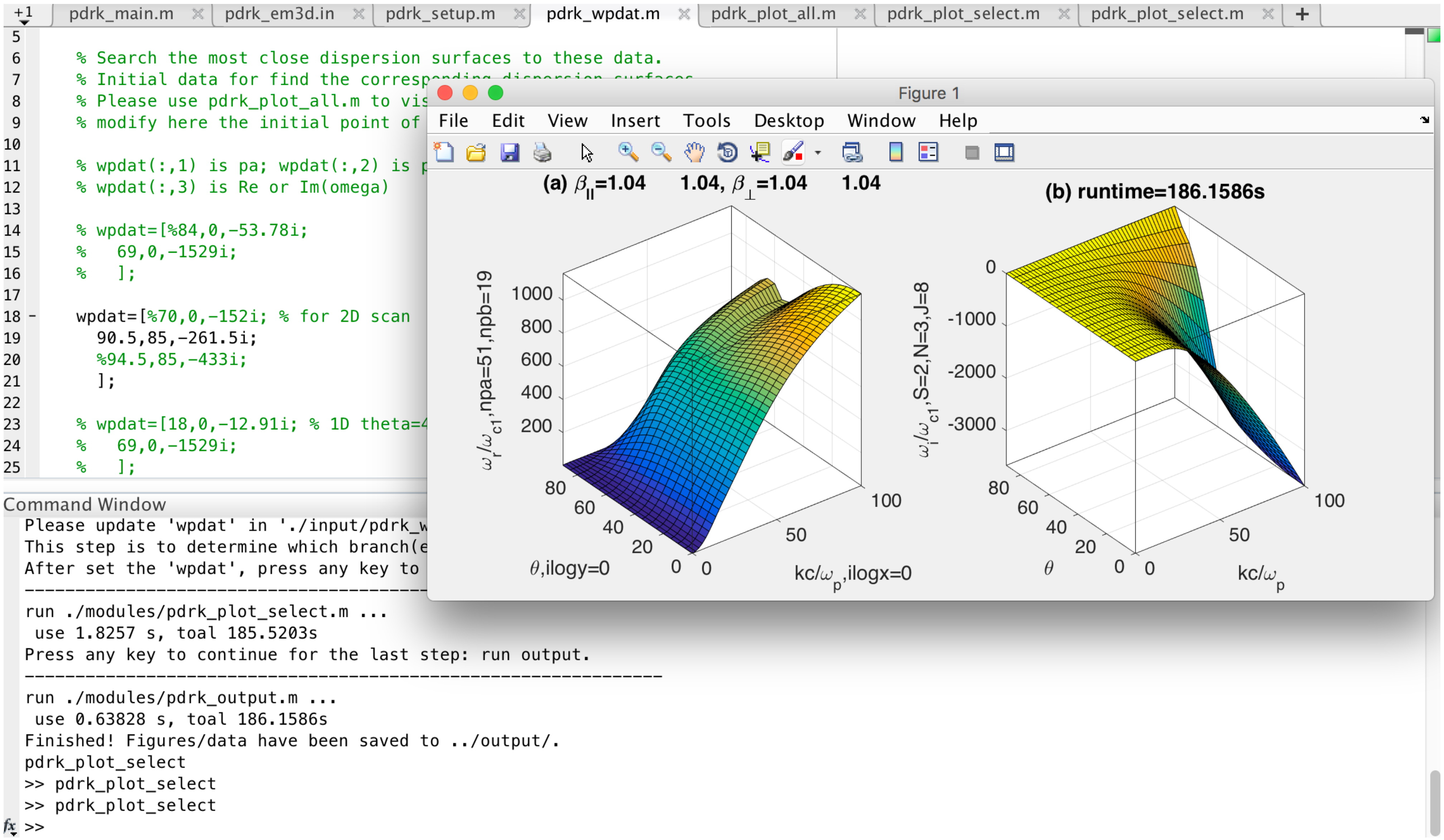
qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
1	1	5.e6	12.94	12.94	1.0	1.0	0.0
-1	5.447e-4	5.e6	12.94	12.94	1.0	1.0	0.0

```
B0=5.0E-9; N=3; J=8; iem=1;

(ipa,ipb) = (1,2) scan 2D (k, theta)

pa=0:2:100;pb=0:5:90;

iout=1;
```

Firstly, theta=45, 1D scan k agrees with whamp.



The 2D scan with proper 'wpdat' gives a nice whistler wave dispersion surface. Give multi 'wpdat' can also plot other dispersion surfaces.



Typical cases 5: ES3D loss cone instability

pdrk.in

qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
-1	5.447e-4	1.e6	1.	1.	1.0	1.0	0.0
-1	5.447e-4	1.e6	5.e2	5.e2	0.005	0.0	0.0

B0= 143.5E-9; N=1; J=8; iem=0;

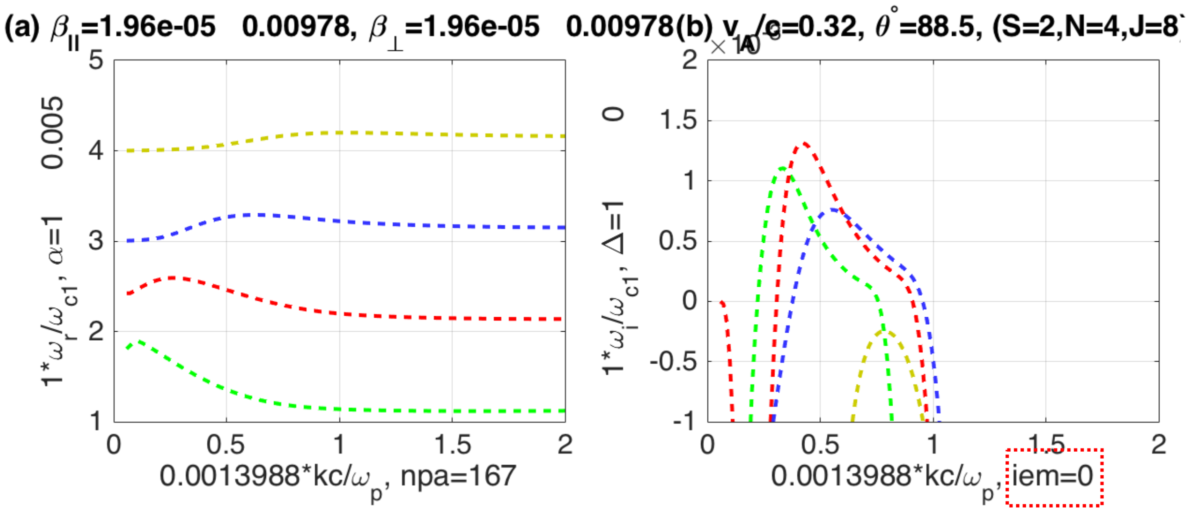
(ipa,ipb) = (1,1) scan k, fixed theta=88.5

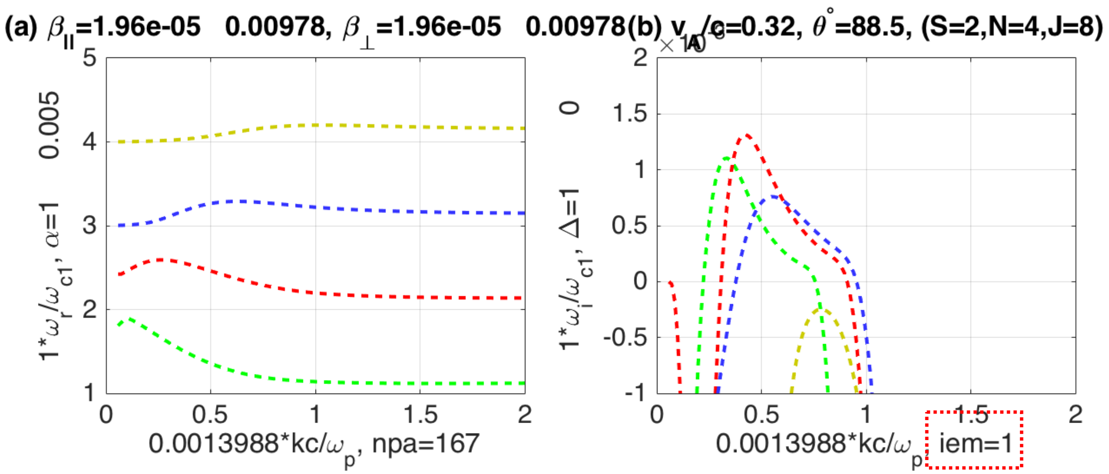
pa=41:10:1700

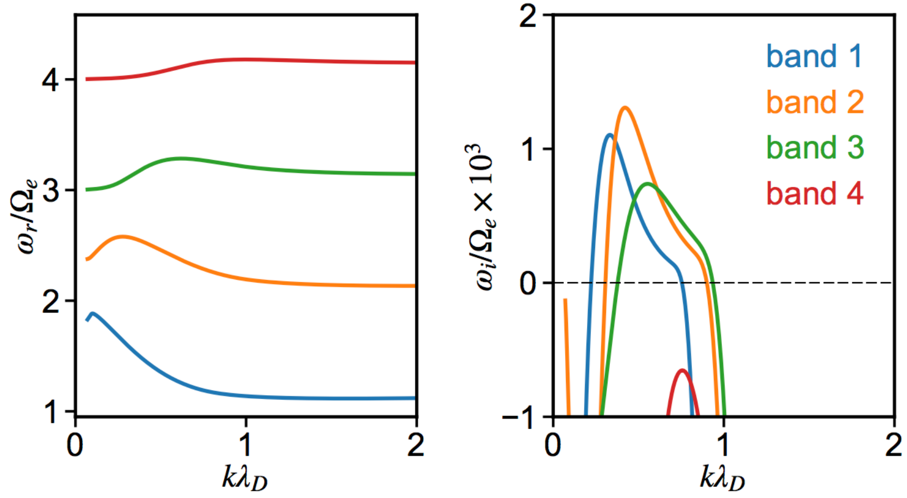
iout=1;

To normalized to cold lambda_De, we have rescaled the k-axis.
rex=abs(lambdaDs(1)/cwp)=0.0013988;

Left: pdrk electrostatic run, iem=0.
Right: pdrk electromagnetic run, iem=1.
Down: HOTRAY electrostatic result from X. Tao et al, 2018 (submitted).







HOTRAY agrees with PDRK very well. And the agreement between iem=0 and iem=1 imply that the mode is indeed electrostatic mode.

Typical cases 6: ES1D beam

pdrk.in

qs(e)	ms(mp)	ns(m^-3)	Tzs(eV)	Tps(eV)	alphas	Deltas	vds/c
-1	5.447e-4	0.9e6	1.	1.	1.0	1.0	0.0
-1	5.447e-4	0.1e6	1.	1.	1.0	1.0	9.8913e-3

B0= 143.5E-9; N=1; J=8; iem=0;

(ipa,ipb) = (1,1) scan k, fixed theta=0

pa=0.1:2:400

iout=1;

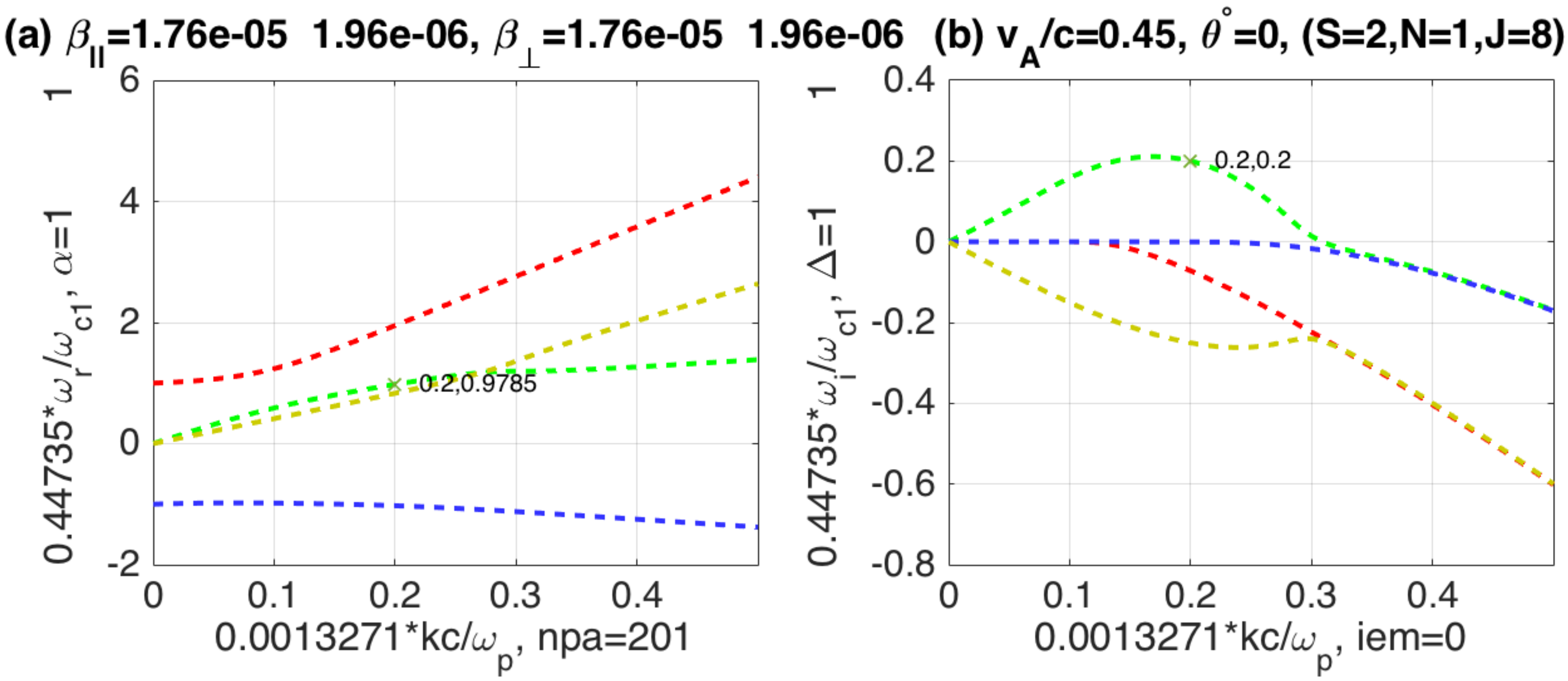
This is default beam test case in Xie2016PST PDRK paper for ES1D version, i.e., kperp=0, nb=0.1n0, ne=n0-nb and vds(2)/vtzs(2)=5.0.

To normalized to omega_pe and lambda_De, we have rescaled the k and omega-axis.

rex=abs(sqrt(1/sum(1./lambdaDs.^2))/cwp)=0.0013271;

rez=abs(wcs(1)/sqrt(sum(wps2)))=0.44735;

Agree well with the original paper, i.e., k*lambda_De=0.2, the most unstable mode omega=0.9785+0.2000i



Enjoy!

If you meet any problems or find pdrk does not agree some benchmarks, please not hesitate to email me (huashengxie@gmail.com), I will long term support this code. Thanks! The suggestions to improve this code are appreciated.

Download: http://hsxie.me/codes/pdrk/

You are welcome to rewrite PDRK to other versions or other languages.

If you use this code, please cite:
[Xie2016] Huasheng Xie and Yong Xiao, PDRK: A General Kinetic Dispersion Relation Solver for Magnetized Plasma, Plasma Science and Technology, 18, 2, 97 (2016). Update/Bugs fixed at http://hsxie.me/codes/pdrk/ or https://github.com/hsxie/pdrk.