New Paradigm for Turbulent Transport Across a Steep Gradient in Toroidal Plasmas

Hua-sheng XIE$^{1,3}$ (谢华生, huashengxie@gmail.com, pku.edu.cn), Yong XIAO$^1$ (肖漷) and Zhihong LIN$^{2,3}$ (林志宏)

$^1$Institute for Fusion Theory and Simulation, Department of Physics, Zhejiang University, Hangzhou 310027, P.R.China
$^2$Department of Physics and Astronomy, University of California, Irvine, California 92697, USA
$^3$Fusion Simulation Center, Peking University, Beijing 100871, China

Max Planck Institut für Plasmaphysik, Garching, Germany, Jun. 22, 2017


Introduction
1. Background: H-mode operation of tokamak

- H-mode (Wagner et al., 1982) is ITER baseline scenario
- Energy stored in H-mode is **twice** or more than L-mode
- Two ‘phases’: L-mode - weak gradient; H-mode - strong gradient

\[
\tau_{E,th}^{ELM} = 0.0562 I^{0.93} B^{0.15} P^{0.69} R^{0.41} \times M^{0.19} \varepsilon^{1.97} \kappa_a^{0.58} K_a^{0.78}
\]

Y. F. Liang, 4TH ITER International Summer School, Austin, Texas USA, 2010
First principle studies of the edge physics still lacking

Physics: a. core - comprehensively studied; **b. edge - current frontier;** c. SOL - more complicated (atom/molecule process)

Existed studies of edge H-mode and ELM

- **Kinetic:** beginning stage (GYRO, GTC, GEM, ···), challenged
- Fluid models: BOUT++, JOREK, ··· → limited kinetic physics
- Simplified models (e.g., ODEs): bifurcation (Itoh-Itoh), prey-predator (Diamond) → qualitative at most

⇒ **We focus on edge kinetic (first principle) physics**
H-mode unknown physics issues

Define heat flux $q_j = \int dv^3 \left( \frac{1}{2} m_j v^2 - \frac{3}{2} T_j \right) \delta v_r \delta f_j \equiv n_j \chi_j \nabla T_j$, $j = i, e$.

- L-H transition is still not fully understood.

- **How will transport coefficient $\chi_j$ changes with $\nabla T_j$ increasing?** Does mixing length ($D \sim l_c^2/\tau_c$) estimation really valid? Or, how to estimate $l_c$ and $\tau_c$? A simplest one $D \sim (\gamma_k/k_\perp^2) \propto \gamma_k$. Taroni-Bohm (Horton2012 book) gives $\chi_e \propto \nabla T_e$.

- Is **zonal flow** still important?

- **How important the mode coupling** can be in the nonlinear evolutions?

Next, we focuses on **edge electrostatic** physics.
Physics understandings in L-mode (weak gradient) still hold in H-mode strong gradient stage?

It is believed that (at least in L-mode stage or core plasmas):

- **Zonal flow** important to reduce transport (eg., Chen01, Waltz08)
- **Mode coupling** important for nonlinear cascading (eg., Lin05, Chen05)
- Larger gradient $\rightarrow$ **larger** transport coefficients

Gyrokinetic simulations
HL-2A H-mode experiments

Typical HL-2A H-mode exp. signal (#14048, from D. F. Kong)

ES: low frequency → this work
2. Nonlinear transport: GTC edge simulation parameters

GTC edge simulation parameters are taken from recent H-mode exp. of HL-2A (#19298, from D. F. Kong)

- $f \sim 80\text{kHz}, \ m \sim 10^{-33}$
- $B_0 = 1.35\ T, \ a = 40\ cm, \ R_0 = 165\ cm, \ q = 2.5 - 3.0, \ s = 0.3 - 1.0$
- $R_0/L_T = 80 - 160, \ \ Te(r) = Ti(r), \ ne(r) = ni(r), \ \eta = L_n/L_T \sim 1.0$

**HL-2A typical L-mode** $R_0L_T^{-1} < 40, \ \text{typical H-mode} \ R_0L_T^{-1} > 80$
Normal turbulent transport understandings in L-mode/weak gradient

Agree with usual understandings / theoretical models

- Stronger gradients in L-mode stage give larger transport coefficients
- Zonal flow can reduce the transport coefficients significantly
Reverse trend of turbulent transport: H-mode/strong gradient

heat conductivity $\chi_j$, particle diffusivity $D_j$

Stronger gradients in H-mode stage give smaller (!!) transport coefficients of particles and energy, though the root mean square of e.s. potential still higher.
A turning point (critical gradient) exists for the reverse trend of the transport coefficients. [Similar to second order phase transition (suggested to add by one of the PRL referee) of Landau1937.]

Eddy sizes - correlation length

Estimate the radial correlation length $l_c$ from the eddy size.

**Strong gradient** ($R_0 L_T^{-1} = 30$) small eddy size. Weak gradient ($R_0 L_T^{-1} = 10$) large eddy size. Assume correlation time $\tau_c$ not change too much $\rightarrow D \sim l_c^2 / \tau_c \propto l_c^2 \rightarrow D \downarrow$.

Next: **Why** stronger gradient has a small eddy size? The formation of the **mode structures** should be examined carefully.
Linear results and theory
3. Linear: two Trapped Electron Mode (TEM) branches

\( n = 20, \ T_e = 200 \text{eV} \) (Right figure: \( R/L_T = 75 \))

Most unstable micro-instabilities under weak and strong gradients are in different branches: (H) \( \omega_r > 10\omega_s, \omega_r \gg \gamma \); (L) \( \omega_r < 3\omega_s, \omega_r < \gamma \).
Various mode structures

Single-$n$ ($n = 5 - 30$)

- (a) weak gradient L-mode parameter gives conventional ballooning structures of TEM in GTC simulation
- (b)-(i) strong gradient H-mode parameters give unconventional structures of TEM.

Mostly unexpected:
- a. anti-ballooning, $|\theta_p| > \pi/2$
- b. multi-peak
Introduction

Gyrokinetic simulations

Linear results and theory

More nonlinear results

Summary

Backup

GTC TEMs

Fourier components $\delta \phi_m(r)$ of TEM

$\delta \phi(r, \theta, \zeta) = e^{in\zeta} \sum_m \delta \phi_m(r)e^{-im\theta}$

Corresponding poloidal cross section mode structures of (a)-(d) are taken from previous (a), (b), (g) and (i), respectively.

- Unconventional mode structures (especially anti-ballooning, $u_m \simeq -u_{m+1}$, i.e., a $180^\circ$ phase shift for neighboring Fourier) can reduce the effective correlation length. We can expect that H-mode can have better confinement.

Strong gradient $|nq - m| > 1 \neq 0$
Model linear theory

- **Model** eigenmode equation for unconventional structure of drift wave

\[
\left[ \rho_i^2 \frac{\partial^2}{\partial x^2} - \frac{\sigma^2}{\omega^2} \left( \frac{\partial}{\partial \theta} + i k_\theta s x \right)^2 - \frac{2\epsilon_n}{\omega} \left( \cos \theta + \frac{i \sin \theta}{k_\theta} \frac{\partial}{\partial x} \right) \right.
\neg \left. - \frac{\omega-1}{\omega+\eta_s} - k_\theta^2 \rho_i^2 \right] \delta \phi(x, \theta) = 0,
\]

\[
\sigma = \frac{\epsilon_n}{(q k_\theta \rho_i)}, \quad \eta_s = 1 + \eta_i, \quad x = r - r_s, \quad \text{poloidal wave number } k_\theta = n q / r
\]

- **1D**: Corresponding 1D equation in ballooning space (normalization: \( \omega_* e \))

\[
\left\{ \frac{\sigma^2}{\omega^2} \frac{d^2}{d\vartheta^2} + k_\theta^2 \rho_i^2 [1 + s^2 (\vartheta - \vartheta_k)^2] + \frac{2\epsilon_n}{\omega} [\cos \vartheta \\
+ s (\vartheta - \vartheta_k) \sin \vartheta] + \frac{\omega-1}{\omega+\eta_s} \right\} \delta \hat{\phi}(\vartheta, \vartheta_k) = 0,
\]

\( \vartheta_k \) ballooning-angle parameter.

- Approximate to Weber equation \( u'' + (b x^2 + a) u = 0 \), eigenvalues \( a(\omega) = i (2l + 1) \sqrt{b(\omega)} \), eigenfunctions \( u(x) = H_l(i \sqrt{b} x) e^{-i b x^2 / 2} \), \( H_l \) is \( l \)-th Hermite polynomial \( (l = 0, 1, 2, \ldots) \), series eigenstates.
1D eigen solutions to drift instability

- **Weak gradient** \((\epsilon_n \equiv L_n/R = 0.5)\), most unstable solution ground state \((a&b)\), conventional structure.

- **Strong gradient** \((\epsilon_n = 0.2)\), most unstable solution not ground state \((c&d)\), unconventional.

- **Condition** \(\epsilon_n < \epsilon_c\), critical gradient parameter \(\epsilon_c\) depends on other parameters.


\[ \text{Eq.(2), series solutions exist.} \quad (s = 0.8, \quad k_\theta \rho_i = 0.4, \quad q = 1.0, \quad \eta_s = 3.0 \text{ and } \vartheta_k = 0) \]

**Linear: Eigenstates jump!!!**
Discussions

- Strong gradient (H-mode) eigen state $l \neq 0$ v.s. weak gradient (L-mode) $l = 0$, indicate different transport behaviors between H-mode and L-mode.

- Unconventional mode structures can reduce the effective correlation length. We can expect that **H-mode can have better confinement**.

- Nonlinear simulations confirm that the transport coefficients decrease with gradient increasing.

Thus ...

Provides some hints to L-H transition and H-mode transport mechanism by first-principle gyrokinetic simulations.

$L \Leftrightarrow H$

Eigenstates jump! vs. ‘phase’ transition?
More nonlinear results
4. Compare with experiment: nonlinear frequency

Diagnose at a fixed point \((r = r_c, \theta = \pi/2, \zeta = 0)\), \(\omega \approx 16\omega_s\)

\[
\begin{align*}
&\text{if } T_e \approx 50\text{eV } \Rightarrow f^{\text{sim.}} \approx 78\text{kHz} + f^{\text{doppler}}, \text{if } |f^{\text{doppler}}| < 10\text{kHz} \\
&\Rightarrow f^{\text{sim.}} \approx f^{\text{exp.}} \approx 80\text{kHz}.
\end{align*}
\]

Nonlinear frequency agrees exp. !!
Nonlinear spectral

Nonlinear evolutions of the poloidal spectral

\( m^{\text{sim.}} \approx 10 - 40 \) vs. \( m^{\text{exp.}} \approx 10 - 33 \), nonlinear poloidal spectral agrees exp. !!

Reverse cascading from high to low \( m \) mode number.
Mode coupling and zonal flow are less important in strong gradient

multi-\(n\) (w/ & w/o zonal flow) vs. single-\(n\)

\[
t = 800t_0, \text{ dominate is } n \approx 20 - 25 \text{ gives } m \approx nq \approx 57;\]
\[
t = 1200t_0, n \approx 15 \text{ gives } m \approx nq \approx 40;\]
\[
t = 2000t_0, n \approx 10 \text{ gives } m \approx nq \approx 26.
\]

Close to multi-\(n\) (previous slide) results, reveal multi-mode-coupling not important for \(m\) downshift as in L-mode [e.g., Wang07, Lang08].
Summary
5. Summary: diagram for new picture of L-H transition

- **H-mode**: Ground (\(l = 0\)) eigenstate
- **L-mode**: Conventional ballooning mode structure
- **Transport 'phases'**: Critical gradient exists
- **Correlation length ↓**: Power input increasing
- **Transport ↓**: Without invoking shear flow or zonal flow
- **Unconventional mode structure**: Micro-instabilities
- **Non-ground (\(l \neq 0\)) eigenstate**: Critical gradient exists

H. S. Xie et al. (IFTS-ZJU & FSC-PKU)
Related works / Backup
6. Related works

- **Gyrokinetic (mainly fixed profiles):**

- **Fluid (profile evolution):**

- **Models:**
  Bifurcation: Itoh & Itoh, 1990s
  Prey-predator: Diamond, 1990s-2010s
Evidences/facts gathering

- Unconventional structures: GEM, GYRO (WangE2012, local), GTC (Fulton2014, global)
- 2D eigen in model (fluid) equation\(^1\): Dickinson2014, XieT2012, McDevitt2015APS (haven’t shown that they are most unstable).
- Local is not conclusive (\(\theta_k \neq \theta_p\)) and previous works have not told what they are, why and how important of them.

A complete picture should include: global, critical gradient, unconventional mode structures, eigenstates jump, consequences & physical understanding

More evidences are gathering, more understandings are required. What about EM (e.g., KBM)?

---

\(^1\)Preliminary global theory: Xie&Li, PoP, 23, 082513 (2016).
Experimental frequency jump before and after L-H transition

- **HL-2A** (Liu2010PoP L-mode, Xie2015 H-mode)
- Experimental frequencies (usually TEMs) jumps from low to high before and after L-H transition have also been reported in **EAST** (Xu2012PoP, Wang2012NF)

More quantitatively and qualitatively experimental evidences are required to support or exclude the new kinetic eigenstates jump picture to L-H transition.
Abstract: First principle gyro-kinetic study of the edge turbulent transport suggests a completed new possible mechanism, without invoking shear flow or zonal flow, for the the low (L) to high (H) confinement modes transition. At H-mode strong gradient the most unstable micro-instabilities are non-ground eigenstates with unconventional mode structures which significantly reduce the effective correlation length and thus reverse the transport trend. Both linear and nonlinear critical gradients exist, which lead **discontinuous jump** as required to explain the L-H transition.

- **The relation of this kinetic picture** to traditional **fluid picture and model theory picture**, where flow shear is usually very important, are **not clear** yet.

- **Our studies are based on first-principle model without artificial parameters** and thus can provide **quantitative outputs** to compare with experiments. How important this new mechanism can be in the past and future experiments can be checked directly.

- **Flow, electromagnetic effects** and **self-consistent evolutions of the profiles** can be considered for the next step to give more quantitative outputs for comparing with experiments.
Gyrokinetic eigen solutions
Gyrokinetic Eigen solutions

Gyrokinetic-Poisson equation (s-α model)\(^2\)

At strong gradient, the most unstable ITG mode transit from \(l = 0\) ground state even mode to \(l \geq 1\) high order ITG modes at \(\varepsilon_n^{-1} R \simeq 50\). The real frequency can transit to electron direction!! \(\rightarrow\) the propagation direction is not a decisive criteria for the experimental diagnosis of turbulent mode at the edge plasmas.

Series higher order ITGs$^3$

For cyclone parameters, $k_\perp \rho_i = 0.4$ and $\epsilon_n = 0.018$. Multi-eigenmodes are shown, where the most unstable modes are around quantum number $l \simeq 2 - 5$.

$^3$See also: M. K. Han, Z. X. Wang, J. Q. Dong and H. R. Du, NF, 2017, 57, 046019.
Gyrokinetic Electromagnetic model

Gyrokinetic Electromagnetic model (with $\delta \phi$, $\delta A_{||}$, remove $\delta B_{||}$, adiabatic electron, $\alpha = 0$), eigen solution

Multi KBMs co-exist, for $l = 0, 1, 2$. The $l = 1$ KBM (i.e., MTM) dominates at $3 \lesssim k_{\theta \rho_i} \lesssim 7$, and the $l = 2$ KBM dominates at $k_{\theta \rho_i} \gtrsim 7$. 
Microtearing mode (MTM) can merely be \( l = 1 \) KBM!

With increasing gradient, i.e., \( \epsilon_n = 0.05, \eta_i = 8.5, l = 3 \) (h&i) KBM can also be found, and the most unstable one is \( l = 2 \) (b&c) under these parameters (\( k_\theta \rho_i = 6.0, \beta_i = 0.05, s = 0.78, q = 1.4, \tau = 1.0 \)).
Scanning of $\eta_e$ for $l = 0, 1, 2$ KBMs with $k_\theta \rho_i = 5.0$. For $\eta_e = 0$, the $l = 0, 1$ KBMs are still unstable, which means that electron temperature gradient is not a must for $l = 1$ KBM (i.e., MTM).

Global effect

Fluid electrostatic model\(^4\)

Steep gradient leads growth rate reduction. \(\epsilon_{-1} = \epsilon_{n0}^{-1} e^{-(r-r_s)^2/\Delta r^2}\)

\(^4\)Xie&Li, PoP, 23, 082513 (2016).
And also twisting (triangle-like) mode structure, due to imag part of $b = k_{\theta}^2 s^2 \omega_{xx}/\omega_{\vartheta_k \vartheta_k}$. Fast particle is not a must!
Other configurations

Strong gradient high order mode also in other configurations, e.g., dipole

Scan $k_{\perp} \rho_i$ (gkd1d code, f90 + mpi)

Against to previous result of $k_{\parallel} \sim 0$ mode dominate, a high order $k_{\parallel} \neq 0$ mode is most unstable at larger $k_{\perp} \rho_i$ for strong gradient $\kappa_n = 18$. New!!